

# MATH 111

## Test 2

April 3, 2006

Name: \_\_\_\_\_

- No books, notes, or calculators are allowed.
- Please show all your work.

1. (8 points) Let  $x \in \mathbb{Z}$ . Prove that if  $x^3 + x^2 + x$  is odd, then  $x$  is odd. What type of proof (direct, by contrapositive, or by contradiction) did you use?

2. (8 points) Prove or disprove.

The sum of two irrational numbers is irrational.

3. (8 points) Prove or disprove.

The sum of a rational number and an irrational number is irrational.

4. (8 points) Prove or disprove.

Let  $A$ ,  $B$ , and  $C$  be sets. If  $A \cup B = A \cup C$ , then  $B = C$ .

5. (6 points) Let  $A = \{1, 2, 3, 4\}$ . Give an example of a relation on  $A$  that is symmetric but not reflexive.

6. (12 points) Let  $R$  be the relation on  $\mathbb{Z}$  defined by  $(a, b) \in R$  iff  $a + b < 5$ .

(a) Is  $R$  reflexive?

(b) Is  $R$  symmetric?

(c) Is  $R$  transitive?

7. (For extra credit, 8 points) Prove or disprove.

The number  $\sqrt{3} + \sqrt{5}$  is irrational.