MATH 111

Practice Test 2

Note: the actual test will consist of five or six questions.

1. This test is primarily on chapters 4-6, however, knowledge of previously covered material may be required. Review all terms, notations, and types of proofs in chapters 0–6.
2. Prove the following statements. Indicate what type of proof (direct, by contrapositive, or by contradiction) you used.
   (a) If n is an integer such that $5|(n-1)$, then $5|(n^3+n-2)$.
   (b) The number $\log_3 2$ is irrational.
   (c) Let $n \in \mathbb{Z}$. If $7n^2+4$ is even, then n is even.
   (d) Let $x \in \mathbb{R}$. If $2x > x^2+x^3$, then $x < 1$.
   (e) Let $m, n \in \mathbb{Z}$. Then $3|(mn)$ if and only if $3|m$ or $3|n$.
   (f) The product of a nonzero rational number and an irrational number is irrational.
   (g) Let $a, b, c \in \mathbb{Z}$. If $a \not|(bc)$, then $a \not|b$ and $a \not|c$.
   (h) Let A and B be sets. Then $A \cap B = \emptyset$ if and only if $(A \times B) \cap (B \times A) = \emptyset$.
3. Use Mathematical Induction to prove the following statements.
   (a) Let $n \in \mathbb{N}$. Then $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + n(n+1) = \dfrac{n(n+1)(n+2)}{3}$.
   (b) Let $f(x) = xe^{-x}$. Then $f^{(n)}(x) = (-1)^n e^{-x}(x-n)$ for every positive integer n.
   (c) Let $n \in \mathbb{N}$. Then $5|(n^5-n)$.