MATH 111

Final Exam

December 18, 2007

Name:____

- No books, notes, or calculators are allowed.
- Please turn off your cell phones.
- Please show all your work.

1. (20 points) Prove that if $A \subset B \cup C$ and $A \cap B = \emptyset$, then $A \subset C$.

2. (20 points) Let $a \in \mathbb{Z}$. Prove that if $3|a^2$, then $3|a$.

3. (20 points) Prove or disprove the following statement:

For any integer a, there exists an integer b such that $b < a$ and $a \equiv b \pmod 2$.

4. (20 points) Prove or disprove the following statement:

The product of a nonzero rational number and an irrational number is irrational.

5. (29 points total) Consider the relation R defined on $\mathbb{Z}$ by $(a, b) \in R$ iff $ab \neq 0$. Determine whether R is

(a) (8 points) reflexive,

(b) (8 points) symmetric,

(c) (8 points) transitive,

(d) (5 points) an equivalence relation.

6. (21 points total) Determine whether the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 + 4x + 9$ is

(a) (8 points) one-to-one,

(b) (8 points) onto,

(c) (5 points) bijective

7. (20 points) Use Mathematical Induction to prove $1 + 3 + 5 + \ldots + (2n - 1) = n^2$ for every positive integer n.

8. (For extra credit, 15 points) How many of the first 1000 natural numbers are solutions of both $a \equiv 2 \pmod 3$ and $a \equiv 3 \pmod 5$?