# MATH 111

## Test 1

September 28, 2010

**Name:**_____

- No books, notes, or calculators are allowed.
- Please show all your work.

1. (4 points) Let S and T be sets. Draw a Venn diagram of $\overline{S \cap T}$.

2. (9 points) Let $A = \{x \in \mathbb{Z} \mid 0 < x < 5\}$ and $B = \{x \in \mathbb{Z} \mid |x| \leq 2\}$, and let $\mathbb{Z}$ be the universal set. Determine the following sets:

   (a) $A \cup B$

   (b) $\overline{A}$

   (c) $\overline{A} \cap B$

3. (12 points) Determine the truth values of the following statements. (Explain your reasoning!)

   (a) $\forall x \in \mathbb{R}\ |x| > 0$

   (b) $\exists x \in \mathbb{Z}\ x^2 = 10$

   (c) $\forall x \in \mathbb{N}\ (x = 5\ \Rightarrow\ 2x \geq 10)$

   (d) $\exists x \in \mathbb{Q}\ (x = 5\ \Leftrightarrow x = 6)$

4. (7 points) Let $P$ and $Q$ be propositions. Prove that the compound propositions $\neg(P \wedge Q)$ and $\neg P \vee \neg Q$ are logically equivalent.

5. (8 points) Which of the following propositions can be proved using a vacuous proof? Prove it (use a vacuous proof).

   - Let $n \in \mathbb{Z}$. Then $5n + 3$ is even if and only if $3n + 6$ is odd.
   - Let $n \in \mathbb{Z}$. If $4n + 6$ is odd, then $7n + 8$ is odd.
   - Let $n \in \mathbb{Z}$. If $3n + 4$ is odd, then $2n + 6$ is even.

6. (10 points) Let $n \in \mathbb{N}$. Prove that $3n - 5$ is even if and only if $n$ is odd.

7. (**For extra credit**, 8 points) Let $P$ and $Q$ denote statements.

   (a) How many non-logically equivalent compound statements in $P$ (i.e. compound statements that contain only one variable, $P$) are there? List all of them.

   (b) How many non-logically equivalent compound statements in $P$ and $Q$ are there? (You do not have to list all of them.)