Math 111 **Name:** ______________________

May 8, 2006 **Test 3**

Attempt all 6 problems. Unsupported work will receive no credit, and partially completed work will receive partial credit. Please turn off your cell phone! Good luck!

1. (8 points) Let R be an equivalence relation defined on a set A containing the elements a, b, c, and d. Prove that if $a\ R\ b$, $c\ R\ d$, and $a\ R\ d$, then $b\ R\ c$.

2. (a) (6 points) Let $f : B \to C$ and $g : C \to D$ be functions such that $g \circ f$ is onto. Prove that g is onto.

(b) (4 points) Give an example of the situation in part (a) in which f is not onto.

3. (8 points) Prove that

$$3 + 7 + 11 + \cdots + (4n - 1) = n(2n + 1)$$

for all $n \geq 1$.

4. Determine whether each of the following functions is one-to-one, onto, neither, or both. Prove your answers.

   (a) (4 points) $f : \mathbb{R} \to \mathbb{R}$, given by $f(x) = \sqrt{x^2 + 7}$.

   (b) (4 points) $f : \mathbb{R} - \{3\} \to \mathbb{R} - \{1\}$, given by $f(x) = \frac{x}{x-3}$.

5. (8 points) A relation R is defined on $\mathbb{Z}$ by $a \; R \; b$ if $5a \equiv 2b \pmod{3}$. Prove that R is an equivalence relation. Determine the distinct equivalence classes.

6. (8 points) Prove that $24 \mid (5^{2n} - 1)$ for every positive integer n.

Extra Credit

(8 points) If $\alpha = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$, prove that $\alpha^{3n} = \underbrace{\alpha \circ \alpha \circ \cdots \circ \alpha}_{3n} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$ for all $n \geq 1$.