MATH 111

Test 2

March 28, 2014

Name:

- Books, any published material, and your own notes are allowed. You may refer to any result in the textbook (feel free to include the result and page numbers).

- Calculators are allowed but are not needed.

- No communication whatsoever is allowed. This includes communication with your classmates, faculty other than the instructor of this class, and any other people e.g. via on-line chat, etc. If you have any questions or comments about this test, please contact your instructor.

- Normally you are expected to be able to complete such an exam in about 50 minutes. However, there is no time limit on this take-home exam. Feel free to spend as much time as needed to ensure that your solutions are complete and correct.

- Please show all your work. Please make your proof complete and detailed.

- The exam must be turned in no later than on Wed, April 2, at 1:00 PM.

1. (10 points) Let $a \in \mathbb{Z}$. Prove that if $9|a^2$, then $3|a$. What type of proof did you use?

2. (10 points) Prove that $\sqrt[3]{2}$ is an irrational number. What type of proof did you use?

3. (10 points) Let A and B be sets. Prove that $(A - B) \cup (A \cap B) = A$.

4. (10 points) Prove that there exists a real number x such that $x^5 + 3x + 2 = 0$.

5. (10 points) Let $n \in \mathbb{N}$. Prove that

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \ldots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}.$$

6. (**For extra credit**, 8 points) Prove that for any positive integer n,

$$1 < \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{3n+1} < 2.$$