3.4. Proof by contrapositive. Suppose $x$ is not even, i.e. is odd. Then $x = 2y + 1$ for some integer $y$, and $7x + 5 = 7(2y + 1) + 5 = 14y + 12 = 2(7y + 6)$. Since $7y + 6$ is an integer, $7x + 5$ is an even integer, therefore is not odd. This proves that if $7x + 5$ is odd then $x$ is even.

3.6. First we will prove that if $5x - 11$ is even then $x$ is odd. We will prove this by contrapositive. Assume $x$ is not odd, i.e. is even. Then $x = 2y$ for some integer $y$, and $5x - 11 = 5(2y) - 11 = 10y - 11 = 10y - 12 + 1 = 2(5y - 6) + 1$. Since $5y - 6$ is an integer, $5x - 11$ is odd, therefore is not even.

Next we prove that if $x$ is odd then $5x - 11$ is even. This part we will prove directly. If $x$ is odd, then $x = 2y + 1$ for some integer $y$, and $5x - 11 = 5(2y + 1) - 11 = 10y - 6 = 2(5y - 3)$. Since $5y - 3$ is an integer, $5x - 11$ is even.

3.8. Lemma. Let $x \in \mathbb{Z}$. If $7x + 4$ is even, then $x$ is even.

Proof of lemma (by contrapositive). Suppose $x$ is not even, i.e. odd. Then $x = 2y + 1$ for some integer $y$, and $7x + 4 = 7(2y + 1) + 4 = 14y + 11 = 14y + 10 + 1 = 2(7y + 5) + 1$. Since $7y + 5$ is an integer, $7x + 4$ is odd, i.e. even.

Proof of the result in the problem. If $7x + 4$ is even, then by the above lemma $x$ is even. Therefore $x = 2y$ for some integer $y$, and $3x - 11 = 3(2y) - 11 = 6y - 11 = 6y - 12 + 1 = 2(3y - 6) + 1$. Since $3y - 6$ is an integer, $3x - 11$ is odd.

3.12. Proof by cases.

Case I. The number $n$ is even. Then $n = 2m$ for some $m \in \mathbb{Z}$, and $n^2 - 3n + 9 = (2m)^2 - 3(2m) + 9 = 4m^2 - 6m + 9 = 4m^2 - 6m + 8 + 1 = 2(2m^2 - 3m + 4) + 1$. Since $2m^2 - 3m + 4 \in \mathbb{Z}$, $n^2 - 3n + 9$ is odd.

Case II. The number $n$ is odd. Then $n = 2m + 1$ for some $m \in \mathbb{Z}$, and $n^2 - 3n + 9 = (2m + 1)^2 - 3(2m + 1) + 9 = 4m^2 + 4m + 1 - 6m - 3 + 9 = 4m^2 - 2m + 7 = 4m^2 - 2m + 6 + 1 = 2(2m^2 - m + 3) + 1$. Since $2m^2 - m + 3 \in \mathbb{Z}$, $n^2 - 3n + 9$ is odd.

3.14. Proof by contrapositive. Suppose that it is not the case that both $x$ and $y$ are odd, i.e. at least one of them is even. Without loss of generality we can assume that $x$ is even. Then $x = 2k$ for some $k \in \mathbb{Z}$, and $xy = (2k)y = 2(ky)$. Since $ky \in \mathbb{Z}$, $xy$ is even, i.e. not odd.
3.16. We will prove the statement by contrapositive, i.e. we will prove that if \( x \) and \( y \) are not of the same parity, then \( 3x + 5y \) is odd.

Case I. If \( x \) is even and \( y \) is odd, then \( x = 2m \) and \( y = 2n + 1 \) for some \( m, n \in \mathbb{Z} \). Then \( 3x + 5y = 3(2m) + 5(2n + 1) = 6m + 10n + 5 = 6m + 10n + 4 + 1 = 2(3m + 5n + 2) + 1 \). Since \( 3m + 5n + 2 \in \mathbb{Z} \), \( 3x + 5y \) is odd.

Case II. If \( x \) is odd and \( y \) is even, then \( x = 2m + 1 \) and \( y = 2n \) for some \( m, n \in \mathbb{Z} \). Then \( 3x + 5y = 3(2m + 1) + 5(2n) = 6m + 3 + 10n = 6m + 10n + 2 + 1 = 2(3m + 5n + 1) + 1 \). Since \( 3m + 5n + 1 \in \mathbb{Z} \), \( 3x + 5y \) is odd.

3.20. Proof by cases.

Case I. The number \( x \) is even. Then \( x = 2y \) for some \( y \in \mathbb{Z} \). Therefore \( 3x + 1 = 3(2y) + 1 = 2(3y) + 1 \) and \( 5x + 2 = 5(2y) + 2 = 10y + 2 = 2(5y + 1) \). Since \( 3y \) and \( 5y + 1 \) are integers, \( 3x + 1 \) is odd and \( 5x + 2 \) is even, so they are of opposite parity.

Case II. The number \( x \) is odd. Then \( x = 2y + 1 \) for some \( y \in \mathbb{Z} \). Therefore \( 3x + 1 = 3(2y + 1) + 1 = 6y + 4 = 2(3y + 2) \) and \( 5x + 2 = 5(2y + 1) + 2 = 10y + 7 = 10y + 6 + 1 = 2(5y + 3) + 1 \). Since \( 3y + 2 \) and \( 5y + 3 \) are integers, \( 3x + 1 \) is even and \( 5x + 2 \) is odd, so they are of opposite parity.

3.22. The converse of the result is proved. The result stated is not proved (because the converse of an implication is not logically equivalent to the implication itself).