Homework 5 (due Wed, March 1)

1. Let $P(x)$ denote “$x > 0$” and let $Q(x)$ denote “$x^2 > 0$” (where $x \in \mathbb{R}$). Determine (and explain!) the truth values of the following propositions:

(a) $\exists x \neg P(x)$
(b) $\forall x (P(x) \lor Q(x))$
(c) $\exists x (P(x) \land Q(x))$
(d) $\forall x (P(x) \Rightarrow Q(x))$
(e) $\exists x (Q(x) \Rightarrow P(x))$
(f) $\forall x (P(x) \Leftrightarrow Q(x))$

2. Are propositions

(a) $\forall x (P(x) \Leftrightarrow Q(x))$ and $(\forall x P(x)) \leftrightarrow (\forall x Q(x))$
(b) $\exists x (P(x) \Leftrightarrow Q(x))$ and $(\exists x P(x)) \leftrightarrow (\exists x Q(x))$

logically equivalent? If so, explain why. If not, give an example of propositional functions $P(x)$ and $Q(x)$ for which one of the propositions is true and the other one is false.

Also do exercises 4.2, 4.4, 4.6, 4.8, 4.12 from the book.