Homework 5 (due Wed, March 1)

1. Let $P(x)$ denote "$x > 0$" and let $Q(x)$ denote "$x^2 > 0$" (where $x \in \mathbb{R}$). Determine (and explain!) the truth values of the following propositions:

   (a) $\exists x \neg P(x)$

   (b) $\forall x (P(x) \vee Q(x))$

   (c) $\exists x (P(x) \wedge Q(x))$

   (d) $\forall x (P(x) \Rightarrow Q(x))$

   (e) $\exists x (Q(x) \Rightarrow P(x))$

   (f) $\forall x (P(x) \Leftrightarrow Q(x))$

2. Are propositions

   (a) $\forall x (P(x) \Leftrightarrow Q(x))$ and $(\forall x P(x)) \Leftrightarrow (\forall x Q(x))$

   (b) $\exists x (P(x) \Leftrightarrow Q(x))$ and $(\exists x P(x)) \Leftrightarrow (\exists x Q(x))$

   logically equivalent? If so, explain why. If not, give an example of propositional functions $P(x)$ and $Q(x)$ for which one of the propositions is true and the other one is false.

Also do exercises 4.2, 4.4, 4.6, 4.8, 4.12 from the book.