4.14. Proof by contrapositive. If \(a < 3m+1\) and \(b < 2m+1\), then since \(a, 3m+1, b, 2m+1\) are integers, it follows that \(a \leq 3m\) and \(b \leq 2m\). Then \(2a+3b \leq 2 \cdot 3m + 3 \cdot 2m = 12m < 12m+1\), so the inequality \(2a+3b \geq 12m+1\) does not hold. Therefore \(2a+3b \geq 12m+1\) implies \(a \geq 3m + 1\) or \(b \geq 2m + 1\).

4.18. Each of \(x\) and \(y\) is either nonnegative or negative. Thus we will consider the following cases.

Case I: \(x \geq 0\), \(y \geq 0\). Then \(xy \geq 0\), and by definition \(|xy| = xy = |x| \cdot |y|\).

Case II: \(x < 0\), \(y < 0\). Then \(xy > 0\), and by definition \(|xy| = xy = (-x)(-y) = |x| \cdot |y|\).

Case III: one of \(x\) and \(y\) is nonnegative and the other one is positive. Without loss of generality we can assume that \(x \geq 0\) and \(y < 0\). Then \(xy \leq 0\), and \(|xy| = -(xy) = x(-y) = |x| \cdot |y|\).

4.20. First we’ll show that \(A \cup B \subset (A - B) \cup (B - A) \cup (A \cap B)\).

Let \(x \in A \cup B\). Then \(x \in A\) or \(x \in B\) (or both). We will consider three cases:

Case I: \(x \in A\) and \(x \notin B\). Then \(x \in A - B\), therefore \(x \in (A - B) \cup (B - A) \cup (A \cap B)\).

Case II: \(x \in B\) and \(x \notin A\). Then \(x \in B - A\), therefore \(x \in (A - B) \cup (B - A) \cup (A \cap B)\).

Case III: \(x \in A\) and \(x \notin B\). Then \(x \in A \cap B\), therefore \(x \in (A - B) \cup (B - A) \cup (A \cap B)\).

Next we’ll show that \((A - B) \cup (B - A) \cup (A \cap B) \subset A \cup B\).

Let \(x \in (A - B) \cup (B - A) \cup (A \cap B)\). Then \(x \in (A - B)\) or \(x \in (B - A)\) or \(x \in (A \cap B)\). So we will consider these three cases.

Case I: \(x \in (A - B)\). Then \(x \in A\), therefore \(x \in A \cup B\).

Case II: \(x \in (B - A)\). Then \(x \in B\), therefore \(x \in A \cup B\).

Case III: \(x \in (A \cap B)\). Then \(x \in A\), therefore \(x \in A \cup B\).

4.22. First we’ll prove that if \(A \cap B = A\), then \(A \subset B\).

Let \(x \in A\). Since \(A \cap B = A\), \(x \in A \cap B\). Therefore \(x \in B\), so \(A \subset B\).

Next we’ll prove that if \(A \subset B\), then \(A \cap B = A\).

To show \(A \cap B = A\), we have to show that \(A \cap B \subset A\) and \(A \subset A \cap B\). The first inclusion holds because if \(x \in A \cap B\), then \(x \in A\). To show the second inclusion, let \(x \in A\). Since \(A \subset B\), by definition \(x \in B\). Then \(x \in A \cap B\).

4.24. Let \(A \cup B \neq \emptyset\). Then there exists an element \(x \in A \cup B\). By definition, \(x \in A\) or \(x \in B\).

If \(x \in A\), then \(A \neq \emptyset\). If \(x \in B\), then \(B \neq \emptyset\).

4.42. The proof contains a mistake. In the third sentence they assume that \(x - 1 = 3q\) and \(y - 1 = 3q\) for some integer \(q\), which implies that \(x = y\). However, \(x\) and \(y\) may not be equal. To correct the proof, different letters must be used, e.g. \(x - 1 = 3q\) and \(y - 1 = 3r\) for some integers \(q\) and \(r\).