Practice Test 2

Note: the actual test will consist of six questions.

1. Review all terms, notations, and types of proofs in chapters 0–6 and sections 7.1 and 7.2.

2. Prove the following statements. Indicate what type of proof (direct, by contrapositive, or by contradiction) you used.

   (a) If n is an integer such that $5|(n-1)$, then $5|(n^3 + n - 2)$.

   (b) The number $\log_3 2$ is irrational.

   (c) Let $n \in \mathbb{Z}$. If $7n^2 + 4$ is even, then n is even.

   (d) Let $m, n \in \mathbb{Z}$. Then $3|(mn)$ if and only if $3|m$ or $3|n$.

   (e) The product of a nonzero rational number and an irrational number is irrational.

   (f) Let $a, b, c \in \mathbb{Z}$. If $a \nmid (bc)$, then $a \nmid b$ and $a \nmid c$.

3. Prove or disprove the following statements.

   (a) There exists a nonzero integer a such that for every real number b, $b^2 \geq a$.

   (b) There exists an integer a such that $a^3 + 2a + 3 = 100$.

   (c) For any sets A and B there exists a set C such that $A \cup C = B \cup C$.

   (d) Let A, B, C, and D be sets such that $A \subset C$ and $B \subset D$. If $A \cap B = \emptyset$, then $C \cap D = \emptyset$.

   (e) Let A, B, C, and D be sets such that $A \subset C$ and $B \subset D$. If $C \cap D = \emptyset$, then $A \cap B = \emptyset$.

4. Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$. Which of the following are relations from A to B?

   (a) $\{(a, 1), (b, 2), (c, 3)\}$

   (b) $\{(1, b), (1, c), (3, a), (4, b)\}$

5. Determine which of the following relations are reflexive; symmetric; transitive.

   (a) Relation R on set $\mathbb{Z}$ defined by $(a, b) \in R$ iff $a + b = 0$.

   (b) Relation R on set $\mathbb{R}$ defined by $(a, b) \in R$ iff $\dfrac{a}{b} \in \mathbb{Q}$.

   (c) Relation R on set $\mathbb{R}$ defined by $(a, b) \in R$ iff $ab > 0$.

   (d) Relation R on set $\mathbb{Z}$ defined by $(a, b) \in R$ iff $a \equiv b \pmod 3$.

   (e) Relation R on set $\mathbb{Q}$ defined by $(a, b) \in R$ iff $a > b$.