MATH 111

Final Exam

December 19, 2007

Name:____

- No books, notes, or calculators are allowed.
- Please turn off your cell phones.
- Please show all your work.

1. (20 points) Let $a \in \mathbb{Z}$. Prove that if $5|a^2$, then $5|a$.

2. (20 points) Prove that there is no smallest positive real number.

3. (20 points) Prove or disprove the following statement:

For any two sets A and B, there exists a set C such that $A \cup C = B \cup C$.

4. (20 points) Prove or disprove the following statement:

For any integer a, there exists an integer b such that $b < a$ and $a \equiv b \pmod{2}$.

5. (29 points total) Consider the relation R defined on $\mathbb{Z}$ by $(a, b) \in R$ iff $ab \geq 0$. Determine whether R is

(a) (8 points) reflexive,

(b) (8 points) symmetric,

(c) (8 points) transitive,

(d) (5 points) an equivalence relation.

6. (21 points total) Determine whether the function $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 2n + 1$ is

(a) (8 points) one-to-one,

(b) (8 points) onto,

(c) (5 points) bijective

7. (20 points) Let $r \in \mathbb{R}$, $r \neq 1$. Use Mathematical Induction to prove $1+r+r^2+\ldots+r^{n-1} = \frac{1-r^n}{1-r}$ for every positive integer n.

8. (For extra credit, 15 points) Does there exist a bijective function from $\mathbb{Q}$ to $\mathbb{Q} - \mathbb{Z}$? Justify your answer.