MATH 111

Practice Test 3

Note: the actual test will consist of five or six questions.

1. This test is primarily on chapters 7-9, however, knowledge of previously covered material may be required. Review all terms, notations, and types of proofs in chapters 0–9.
2. Prove or disprove the following statements.
   (a) There exists a nonzero integer a such that for every real number b, $b^2 \geq a$.
   (b) There exists an integer a such that $a^3 + 2a + 3 = 100$.
   (c) For any integer a there exists an integer b such that $b^2 = a$.
   (d) The sum of any two positive irrational numbers is irrational.
   (e) Any irrational number is the sum of an irrational number and a positive rational number.
   (f) For any sets A and B there exists a set C such that $A \cup C = B \cup C$.
   (g) Let A, B, C, and D be sets such that $A \subset C$ and $B \subset D$. If $A \cap B = \emptyset$, then $C \cap D = \emptyset$.
   (h) Let A, B, C, and D be sets such that $A \subset C$ and $B \subset D$. If $C \cap D = \emptyset$, then $A \cap B = \emptyset$.
3. Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$. Which of the following are relations from A to B or relations from B to A? Which of them are functions?
   (a) $\{(a, 1), (b, 2), (c, 3)\}$
   (b) $\{(1, b), (1, c), (3, a), (4, b)\}$
4. Determine which of the following relations are reflexive; symmetric; transitive. Which of them are equivalence relations? For those that are, describe the distinct equivalence classes.
   (a) Relation R on set $\mathbb{Z}$ defined by $(a, b) \in R$ iff $a + b = 0$.
   (b) Relation R on set $\mathbb{R}$ defined by $(a, b) \in R$ iff $\frac{a}{b} \in \mathbb{Q}$.
   (c) Relation R on set $\mathbb{R}$ defined by $(a, b) \in R$ iff $ab > 0$.
   (d) Relation R on set $\mathbb{Z}$ defined by $(a, b) \in R$ iff $a \equiv b \pmod 3$.
   (e) Relation R on set $\mathbb{Q}$ defined by $(a, b) \in R$ iff $a > b$.
5. Determine which of the following functions are one-to-one; onto; bijective.
   (a) $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 5n^2 + 2$.
   (b) $f : \mathbb{N} \to \mathbb{R}$ defined by $f(n) = \frac{1}{n}$.
   (c) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \begin{cases} \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

(d) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3 - x$.