MATH 114

Final Exam

December 13, 2004

Name:____

- No books, notes, or calculators are allowed.
- Please provide detailed explanations. When necessary, provide examples and counterexamples.

1. (12 points) Is the function $f(x) = x^2$ from $\mathbb{N}$ to $\mathbb{N}$

   (a) one-to-one?

   (b) onto?

2. (12 points) Use Mathematical Induction to prove that $2^n < n!$ for every positive integer n with $n \geq 4$.

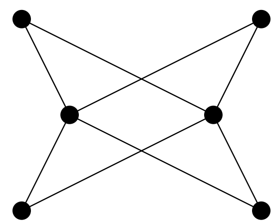
3. (12 points) Let $P(x, y)$ denote the proposition $y = x + 5$ where x and y are positive integers. Determine the truth value of the following propositions.

   (a) $\forall x \exists y P(x, y)$

   (b) $\forall y \exists x P(x, y)$

   (c) $\exists y \forall x P(x, y)$

4. (12 points) Consider the following graph.



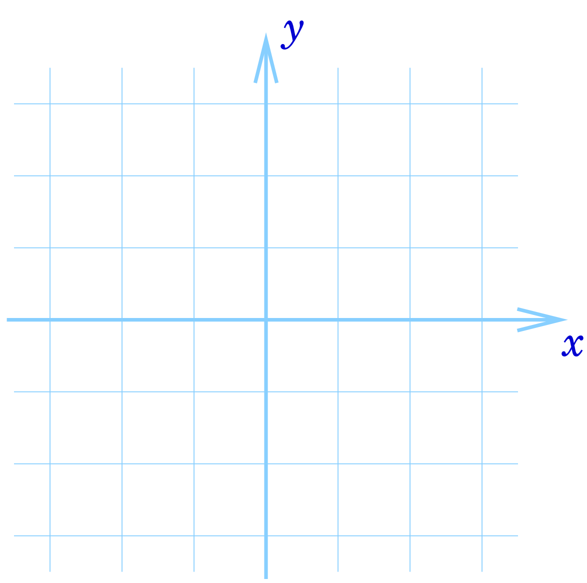
(a) How many vertices does this graph have?

(b) How many edges does this graph have?

(c) Is this graph bipartite?

5. (10 points) How many different strings can be made by reordering the letters of the word *SUCCESS*?

6. (10 points) Draw the graph of $f(x) = \lceil x^2 - 2 \rceil$.



7. (10 points)

   (a) If $a|c$ and $b|c$, does a necessarily divide b?

   (b) If $a|b$ and $b|c$, does a necessarily divide c?

8. (10 points)

(a) Show that the relation $R = \{(a, b) \mid \lfloor a \rfloor = \lfloor b \rfloor\}$ on the set of real numbers is an equivalence relation.

(b) How many equivalence classes are there for this equivalence relation? Describe them.

9. (12 points) Thirteen small insects are placed inside a 1×1 square. Show that at any moment there are at least four insects which can be covered by a single disk of radius $\frac{2}{5}$.

For extra credit:

10. (10 points) Prove that infinitely many Fibonacci numbers are divisible by 10.