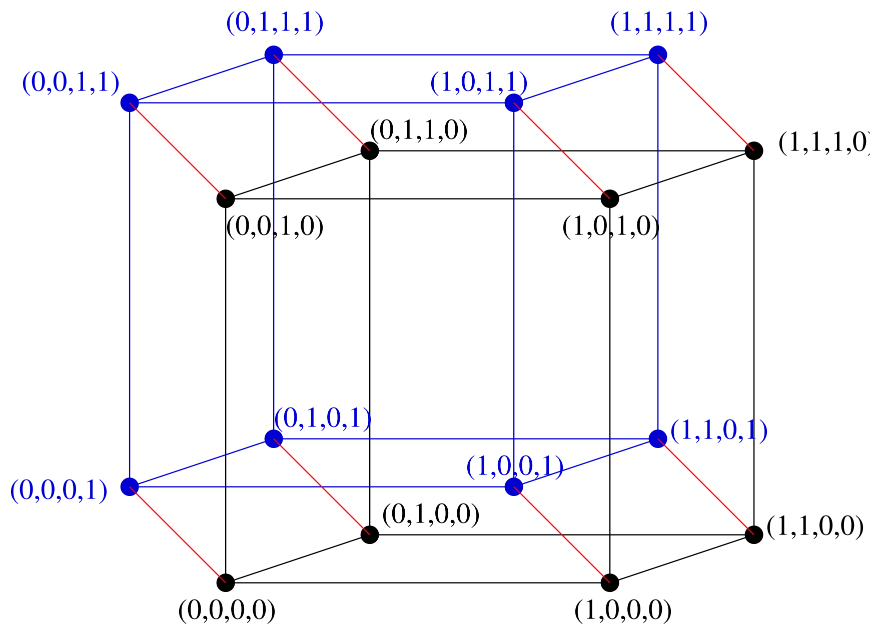
MATH 114 Homework 15 - Selected solutions

8.2 # 18(f) (optional)

Let the cube be the set of points (x_1, x_2, x_3, x_4) in $\mathbb{R}$ such that $0 \leq x_i \leq 1$ for each i. Then the set of vertices is the set of ordered 4-tuples of zeros and ones. There are 16 vertices. Two vertices are connected if and only if their coordinates differ in exactly one position. The set of vertices with $x_4 = 0$ and edges connecting these vertices form Q_3 shown in black. The vertices with $x_4 = 1$ and edges connecting these form another copy of Q_3, shown in blue. Finally, red edges connect vertices in the black Q_3 and the blue Q_3 whose coordinates x_1, x_2, and x_3 are the same (but x_4 are different).



8.2 #24 For which values of n are these graphs bipartite?

(a) K_n

K_1 is bipartite if we allow one of the sets (V_1 or V_2 using the notation in definition 5 on page 550) to be empty (the book does).

K_2 is bipartite because we can let one vertex be in V_1 and the other vertex to be in V_2.

K_n for $n \geq 3$ is not bipartite: choose any 3 vertices. They all are pairwise connected, therefore there is no way to partition them into two disjoint sets V_1 or V_2 such that there are no edges within V_1 and no edges within V_2.

(b) C_n

C_n is bipartite if and only if n is even. Label the vertices by 1, 2, ... consecutively along the cycle. If vertex 1 is in V_1 then vertex 2 must be in V_2, vertex 3 must be in V_1, vertex 4 must be in V_2, and so on. All vertices with odd number are in V_1 and all vertices with even number are in V_2. The last vertex is in V_1 if n is odd and it is in V_2 if n is even. But it is connected to vertex 1. We see that if n is odd, the graph is not bipartite, and if n is even, the graph is bipartite.

(d) Q_n **(optional)**

Q_n is bipartite for any n. Let V_1 consist of all vertices whose sum of coordinates is odd and let V_2 consist of all vertices whose sum of coordinates is even. Two vertices in Q_n are connected if and only if their coordinates differ in only one position, therefore the sums of their coordinates have different parity, so they are in different sets.