MATH 114 Practice problems for Final Exam

Reminder: the **final exam** is on Monday, December 13 from 11am - 1am.

Office hours: Thursday, December 9 from 1 - 5pm; Friday, December 10 from 11am - 3pm.

Try to do the following problems on your own first. If you are having difficulty with a problem, review the indicated section. Solutions to these problems will be posted on the course web page. In addition to doing these problems, I recommend you to review all past tests, practice problems for the tests, and homework problems.

1. (1.1, 1.2) Each inhabitant of a remote village always tells the truth or always lies. A villager will only give a "yes" or "no" response to a question a tourist asks. Suppose you are a tourist visiting this area and come to a fork in the road. One branch leads to the ruins you want to visit; the other branch leads deep into the jungle. A villager is standing at the fork in the road. Explain how the villager's answer to your question "if I were to ask you whether the right branch leads to the ruins, would you answer yes?" will tell you which road to take. Which logic law are you using?
2. (1.2) Use a truth table to show that propositions $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.
3. (1.3) Translate the statement
$$\forall x(C(x) \vee \exists y(C(y) \wedge F(x,y)))$$
into English, where $C(x)$ is "x has a computer", $F(x,y)$ is "x and y are friends", and the domain for both x and y is the set of all students in Fresno.
4. (1.4) Let $P(x,y)$ denote the proposition "$x < y$" where x and y are real numbers. Determine the truth values of
   (a) $\exists x \exists y P(x,y)$,
   (b) $\forall x \exists y P(x,y)$,
   (c) $\exists x \forall y P(x,y)$,
   (d) $\forall x \forall y P(x,y)$.
5. (1.5) Show that $\sqrt[3]{25}$ is irrational.
6. (1.6) If $A = \{1,2,3\}$, $B = \{a,b\}$, and $C = \emptyset$, list all elements of $A \times B$ and $A \times C$.
7. (1.7) In a group of 20 people 14 speak Spanish, 9 speak French, and 4 speak German. Every person speaks at least one of these languages but only one person speaks all three. How many people speak exactly 2 of these languages?
8. (1.8) Draw the graphs of $f(x) = \lfloor 2x+1 \rfloor$ and $g(x) = \lceil 3 - x^2 \rceil$. If we consider $f(x)$ and $g(x)$ as functions from $\mathbb{R}$ to $\mathbb{Z}$, are they one-to-one? Are they onto?
9. (2.4)
   (a) If $a|b$ and $a|c$, does a necessarily divide $b+c$?
   (b) If $a|c$ and $b|c$, does $a+b$ necessarily divide c?
10. (2.5)
   (a) Convert 12 from decimal notation to binary notation.
   (b) Convert 68DAC0 from hexadecimal (base 16) notation to decimal notation.

11. (2.6) Find the greates common divisor d of $a = 46$ and $b = 32$, and integer numbers x and y such that $xa + yb = d$.
12. (3.1) Show that 3 divides $a^2 + b^2$ iff 3 divides both a and b (where a and b are integers).
13. (3.2) Find the value of the sum $\sum_{i=1}^{100} (2i^2 + 2^i)$.
14. (3.3) Prove that $1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{n^2} < 2 - \frac{1}{n}$ whenever n is a positive integer greater than 1.
15. (3.4) Let $f_1(x) = 2x + 1$ and $f_n = f_1 \circ f_{n-1}$. Compute f_n for some small values of n. Notice the pattern. Write a formula for f_n and prove it using Mathematical Induction.
16. (4.1) A witness to a hit-and-run accident tells the police that the license plate of the car in the accident, which contains three letters followed by three digits, starts with the letters AS and contains both the digits 1 and 2. How many different license plates can fit this description?
17. (4.2) How many people are needed to guarantee that at least six of them have the same sign of the zodiac?
18. (4.3) There are 10 projects and 5 groups of people.
    (a) How many ways are there to choose 5 projects out of 10?
    (b) How many ways are there to choose 5 projects out of 10 and assign them to the 5 groups so that each group is assigned one project?
    (c) How many ways are there to assign all 10 projects to the 5 groups so that each group is assigned two projects?
19. (4.4) Show that if n is a positive integer then $\sum_{k=0}^{n} 3^k \binom{n}{k} = 4^n$.
20. (4.5) How many ways are there to travel in xyz space from the origin $(0, 0, 0)$ to the point $(3, 4, 5)$ by taking steps one unit in the positive x direction, one unit in the positive y direction, or one unit in the positive z direction? (Moving in the negative x, y, or z direction is prohibited.)
21. (7.1) Consider the relation R on the set of natural numbers defined by $(a, b) \in R$ iff $\log_a b \in \mathbb{Z}$. Is R reflexive? Symmetric? Antisymmetric? Transitive?
22. (7.5) Show that the relation R on the set of functions from $\mathbb{R}$ to $\mathbb{R}$ defined by $(f(x), g(x)) \in R$ iff $f(0) = g(0)$ is an equivalence relation. Describe its equivalence classes.
23. (8.1)
    (a) There are 9 counties in Sikinia. There are no "four corners" points (like Arizona, Colorado, New Mexico, and Utah). Each county counted the number of neighboring counties. The numbers are 5, 4, 4, 4, 3, 3, 2, 2, and 2. Prove that at least one county made a mistake.
    (b) If the numbers are actually 6, 4, 4, 4, 3, 3, 2, 2, and 2, draw a possible map.
    (c) For the above map, draw a graph with vertices representing counties where two vertices connected if and only if the correponding counties are neighbors.
24. (8.2) There are 10 men and 10 women attending a dance. Each man knows exactly two women and each woman knows exactly two men. Show that after suitable pairing, each man can dance with a woman he knows.