MATH 114

Test 1

October 1, 2004

Name:__________________________________________

• No books, notes, or calculators are allowed.

• Please show all your work.
1. (5 points) Show that \((\neg q \land (p \rightarrow q)) \rightarrow \neg p\) is a tautology.
2. (10 points) Let $P(x, y)$ be the statement $x^2 < y$ where both $x$ and $y$ are real numbers. Determine the truth values of the following propositions. Give examples and explanations to support your answers.

(a) $P(3, 4)$

(b) $\forall x \forall y P(x, y)$

(c) $\exists y \exists x P(x, y)$

(d) $\forall x \exists y P(x, y)$

(e) $\exists x \forall y P(x, y)$
3. (5 points) Prove that the sum of two odd numbers is even.

4. (10 points) Let $S = \{1, 2, 3, 4\}$ and $T = \{2, 4, 5\}$. Find the following:
   (a) The cardinality of $S$.
   (b) $S \cup T$
   (c) $S \cap T$
   (d) $S - T$
   (e) How many elements are there in $S \times T$?
5. (12 points) Which of the following functions \( \mathbb{R} \rightarrow \mathbb{R} \) are one-to-one? onto? Explain.

(a) \( f(x) = -x + 2 \)

(b) \( f(x) = e^x \)

(c) \( f(x) = x^4 \)
6. (8 points) Sketch the graphs of \( f(x) = [1 - x] \) and \( g(x) = [-1 - x] \).