MATH 114

Test 1

October 1, 2004

Name:

- No books, notes, or calculators are allowed.

- Please show all your work.

1. (5 points) Show that $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology.

2. (10 points) Let $P(x, y)$ be the statemnt $x^2 < y$ where both x and y are real numbers. Determine the truth values of the following propositions. Give examples and explanations to support your answers.

   (a) $P(3, 4)$

   (b) $\forall x \forall y P(x, y)$

   (c) $\exists y \exists x P(x, y)$

   (d) $\forall x \exists y P(x, y)$

   (e) $\exists x \forall y P(x, y)$

3. (5 points) Prove that the sum of two odd numbers is even.

4. (10 points) Let $S = \{1, 2, 3, 4\}$ and $T = \{2, 4, 5\}$. Find the following:

(a) The cardinality of S.

(b) $S \cup T$

(c) $S \cap T$

(d) $S - T$

(e) How many elements are there in $S \times T$?

5. (12 points) Which of the following functions $\mathbb{R} \to \mathbb{R}$ are one-to-one? onto? Explain.
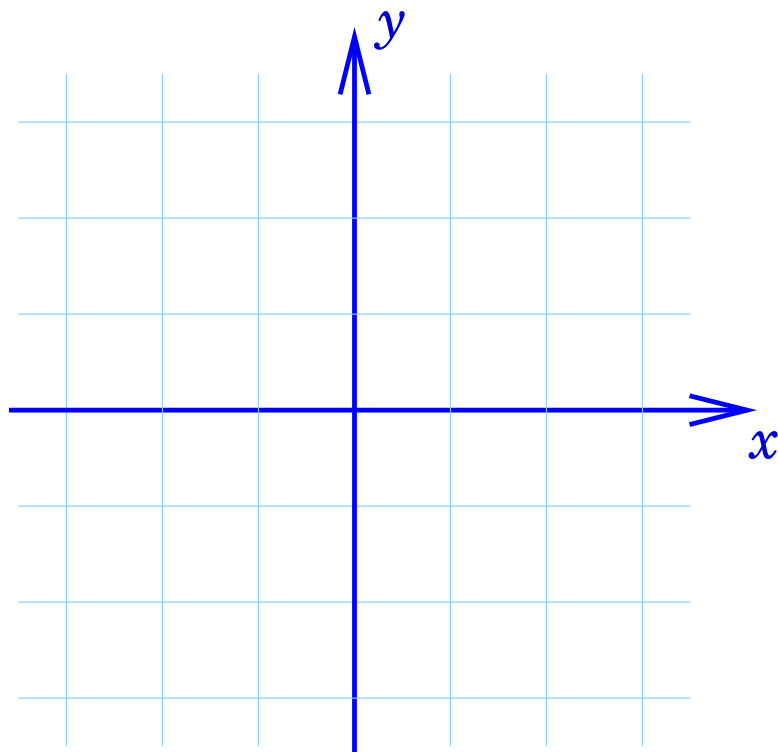
   (a) $f(x) = -x + 2$

   (b) $f(x) = e^x$

   (c) $f(x) = x^4$

6. (8 points) Sketch the graphs of $f(x) = \lfloor 1 - x \rfloor$ and $g(x) = \lceil -1 - x \rceil$.

$y = \lfloor 1 - x \rfloor$

$y = \lceil -1 - x \rceil$