

# MATH 114

## Test 2

November 5, 2004

Name: \_\_\_\_\_

- No books, notes, or calculators are allowed.
- Please show all your work.
- There are 5 problems. Each problem is worth 10 points.

1. (a) Find the inverse of 5 modulo 8.

(b) Solve the congruence  $5x \equiv 7 \pmod{8}$ .

2. Solve the system

$$\begin{cases} x \equiv 0 \pmod{3} \\ x \equiv 2 \pmod{7} \\ x \equiv 1 \pmod{10} \end{cases}$$

3. Prove that for any nonnegative integer  $n$ ,  $n^3 + 5n$  is divisible by 6.

4. Use Mathematical Induction to prove that for any positive integer  $n$ ,

$$1 \cdot 2 + 2 \cdot 3 + \dots + n(n+1) = \frac{1}{3}n(n+1)(n+2).$$

5. A password must contain 8 characters. Each character can be either a digit or a letter. The password must contain at least one digit and at least one letter. How many different passwords are possible?