MATH 114

Test 3

December 6, 2004

Name:_____

- No books, notes, or calculators are allowed.
- Please provide detailed explanations.

1. (8 points) Show that the relation $R = \{(a, b) \mid a - b \text{ is an even integer}\}$ is an equivalence relation and describe the equivalence class of a real number r. What is the equivalence class of 1.5?

2. (5 points) How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 20$$

where x_1, x_2, x_3, x_4, x_5 are nonnegative integers?

3. (14 points total)

(a) Draw K_5.

(b) Draw $K_{3,4}$.

(c) How many vertices and how many edges does $K_{n,m}$ have?

4. (6 points) What is the coefficient of x^5y^{10} in the expansion of $(2x - y)^{15}$?

5. (7 points) How many strings of 8 upper case letters from the English alphabet contain exactly two As and exactly three Bs?

6. (10 points) 40 different numbers are chosen from the set $\{1, 2, \ldots, 100\}$. Show that there are at least 4 different pairs of these numbers with the same sum.