Dirichlet's box principle

Problems

Prove the following statements.

1. If there are n persons present in a room, and every person knows at least one other person, then among them there are 2 persons who have the same number of acquaintances.
2. Suppose that 5 lattice points are chosen in the plane lattice. Then we can choose 2 of these points such that the segment joining these 2 points passes through another lattice point.
3. From 52 positive integers, we can select two such that their sum or difference is divisible by 100. Is the assertion also valid for 51 positive integers?
4. Let every block of a 3×7 checkerboard be colored either black or white. Prove that in whichever way you color the checkerboard, it contains a rectangle consisting of more than one row and more than one column whose four corners have the same color.
5. Let twenty pairwise distinct positive integers be all less than 70. Then among their pairwise differences there are four equal numbers.
6. Suppose that fifty-one small insects are placed inside a square of side 1. Then at any moment there are at least three insects which can be covered by a single disk of radius $1/7$.
7. Let n be a positive integer which is not divisible by 2 or 5. Prove that there is a multiple of n consisting entirely of ones.
8. From 11 infinite decimals, we can select two numbers a and b so that the decimal representation of $a - b$ is finite or has infinitely many zeros.