

Dirichlet's box principle

Theory

If $n + 1$ pearls are put into n boxes, then at least one box has more than one pearl.

Dirichlet's box principle is often called **the Pigeonhole Principle** and is formulated as follows.

Suppose there are n pigeonholes in the tree, and there are $n + 1$ pigeons flying into these n holes. Then there is at least one hole containing more than one pigeons.

More formally and more generally, this principle can be formulated in the following way.

If the cardinality of a set S is bigger than the cardinality of a set T , and f is a function from S to T , then f is not one-to-one.

$$|S| > |T|$$

$$S \xrightarrow{f} T$$

e.g.

$$\begin{aligned} \{n + 1 \text{ pearls}\} &\longrightarrow \{n \text{ boxes}\} \\ \{n + 1 \text{ pigeons}\} &\longrightarrow \{n \text{ pigeonholes}\} \end{aligned}$$

Examples.

1. Among 13 persons, there are two born in the same month.
2. If $qs + 1$ pearls are put into s boxes, then at least one box has more than q pearls.
3. Nobody has more than 300,000 hairs on his head. The capital of Sikinia has 300,001 inhabitants. Can you assert with certainty that there are two persons with the same number of hairs on their heads?
4. Let $\{a_1, a_2, \dots, a_{n+1}\}$ be numbers from the set $\{1, 2, \dots, 2n\}$. Then one of the a_i 's is divisible by another.