Math 145 **Fall 2003**

Graphs

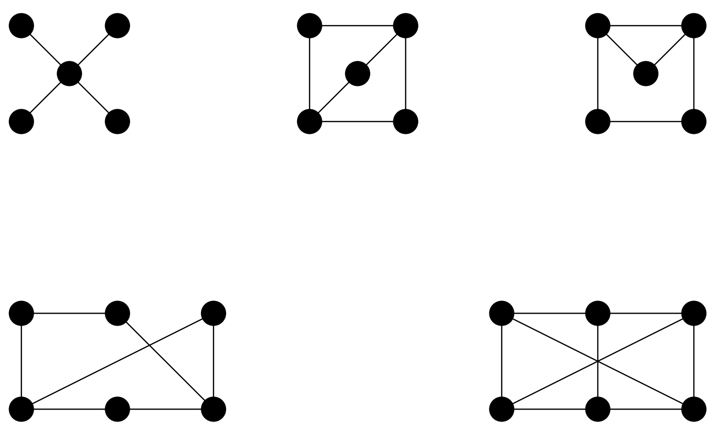
Problems

1. Can a graph have 6 vertices of degrees 4, 3, 3, 2, 2, and 1?

2. How many edges does a graph have if it has vertices of degrees 4, 3, 3, 2, 2? Draw such a graph.

3. Find the number of vertices and edges in K_n and $K_{n,m}$.

4. Determine which of the following graphs are bipartite:



5. Which of the above graphs have

   (a) an Euler path?

   (b) an Euler cycle?

   (c) a Hamilton path?

   (d) a Hamilton cycle?

6. Find a necessary and sufficient condition for a graph to have an Euler path but not an Euler cycle.

7. For which values of n does K_n have

   (a) an Euler path?

   (b) a Hamilton path?

8. There are 17 scientists who communicate with each other to discuss some problems. In their communications, assume they discuss only three topics. Prove that there are at least 3 scientists who are discussing the same topic.

9. A knight's tour is a sequence of legal moves by a knight starting at some square of a chessboard and visiting each square exactly once. A knight's tour is called reentrant if there is a legal move that takes the knight from the last square of the tour back to where the tour began.

    (a) Draw the graph that represents the legal moves of a knight on a 3×4 chessboard.

    (b) Show that there is no reentrant tour on a 3×4 chessboard.

    (c) Find a non-reentrant tour on a 3×4 chessboard.

10. There are 10 men and 10 women at a dance. Every man knows exactly 2 women and every woman knows exactly 2 men. Prove that after suitable pairing, every man can dance with a woman he knows.

11. (a) Prove that in a finite simple graph having at least 2 vertices there are always two vertices with the same degree.

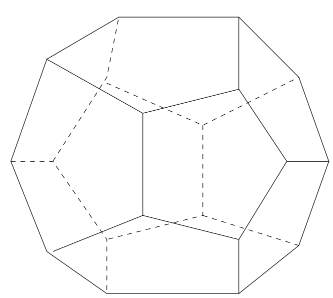
    (b) Does the above hold for graphs with loops (but no multiple edges)?

    (c) Does the above hold for graphs with multiple edges (but no loops)?

12. Hamilton's "Round the World" puzzle: does the dodecahedron (shown below) have

    (a) a Hamilton path?

    (b) a Hamilton cycle?



13. Nine mathematicians met at an international conference. They found that among any 3 of them there are 2 that have a language in common. If every mathematician speaks at most 3 languages, prove that at least three of the mathematicians can speak the same language.