**Math 145** **Fall 2003**

# Homework 11

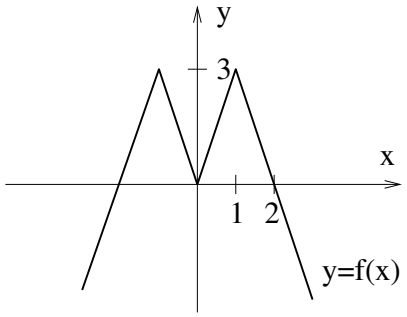
## Working backwards

**Due 21 November 2003**, 5 points each:

1. Let $a = 96$ and $b = 54$. Find the greatest common divisor $d$ of $a$ and $b$, and use Euclid's algorithm to find $x$ and $y$ such that $xa + yb = d$.

2. Find $a$ and $b$ such that in Euclid's algorithm $r_7 = (a, b)$. Write out all the divisions.

3. Find a formula for the function whose graph is shown below.




4. Suppose you are writing a calculus book. You want to find a few cubic polynomials $f(x) = ax^3 + bx^2 + cx + d$ (preferably with integer coefficients) whose critical numbers are integers. (Recall that a critical number is a value of $x$ at which the derivative is equal to 0.) How would you find such polynomials? Use your strategy to find a couple of polynomials.

5. Two players play the following game.

   - Turns alternate.
   - At each turn, a player removes 1, 2, 3, or 4 counters from a pile that had initially 27 counters.
   - The game ends when all counters have been removed.
   - The player who takes the last counter loses.

   Find a winning strategy for one of the players.

**Extra credit:** Starting with 2, 0, 0, 3, we construct the sequence 2, 0, 0, 3, 5, 8, 6, ..., where each new digit is the mod 10 sum of the preceding four terms. Will the 4-tuple 0, 4, 0, 7 ever occur?