# Homework 12

Calculus

Due 1 December 2003, 5 points each:

1. Find the n-th derivative of $f(x) = \dfrac{1}{x^2 + x}$.
   Hint: use the partial fraction decomposition. Recall that since $x^2 + x = x(x+1)$, the partial fraction decomposition has the form $\dfrac{A}{x} + \dfrac{B}{x+1}$.

2. Sketch the region $S = \{(x, y) \mid |x| \geq 1, \ |y| \geq 2, \ x^2 + y^2 \leq 9\}$ and find its area.

3. Find a such that the area of the region bounded by the line $y = ax$ and the parabola $y = x^2$ is equal to 1.

4. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^{2n+1}} = \frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^5} + \frac{1}{2^7} + \ldots$
   Hint: e.g. factor out $\dfrac{1}{2}$, and notice that $2^{2n} = 4^n$.

5. The figure below shows a circle with radius 1 inscribed in the parabola $y = x^2$. Find the center of the circle.
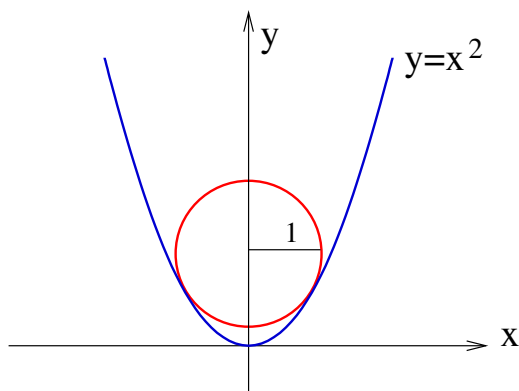
Extra credit: Find the volume of a 4-dimensional unit ball.