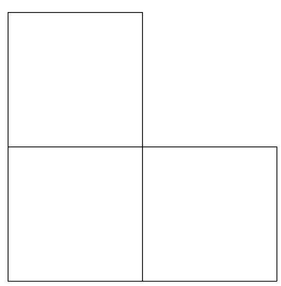
Math 145 **Fall 2003**

Homework 2

Mathematical Induction

Do these by 12 September 2003, 5 points each:

1. Prove the following identity for Fibonacci numbers: $F_{n-1}F_{n+1} = F_n^2 + (-1)^n$.
2. Every road in Sikinia is one-way. Every pair of cities is connected by exactly one direct road. Show that there exists a city which can be reached from every other city either directly or via at most one other city.
3. A map can be properly colored (see problems done in class for definition of a proper coloring) with two colors iff ("iff" means "if and only if") all of its vertices have even degree.
4. Prove that $1 < \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{3n+1} < 2$.
5. If one square of a $2^n \times 2^n$ chessboard is removed, then the remaining board can be covered by L-trominoes, i.e. the figures consisting of 3 squares as shown below.



(You can choose which square you want to remove.)

For extra credit, can be submitted at any time during the semester:

Find the determinant of the $n \times n$ matrix M_n with entries $m_{ij} = \begin{cases} a \text{ if } i = j \\ b \text{ if } |i-j| = 1 \\ 0 \text{ otherwise} \end{cases}$ for arbitrary a and b. Suggestion: Find a recursive equation, prove it using Mathematical Induction, and then find an explicit formula for the determinant of such $n \times n$ matrix. We will see later in this course how to find explicit formulas from (linear) recursive equations. Ask me for more information if you would like to do a project on this.