Homework 3

Logic and types of proofs

Due 19 September 2003, 5 points each:

1. Show that $p \to q$ and $\neg q \to \neg p$ are logically equivalent.
2. Let $Q(x, y)$ be the statement "$x + y = x - y$", and the domain for both variables is the set of integers. Find the truth values of the following statements. Explain.
   (a) $Q(2, 0)$
   (b) $\forall y Q(1, y)$
   (c) $\forall x \exists y Q(x, y)$
   (d) $\forall y \exists x Q(x, y)$
   (e) $\exists y \forall x Q(x, y)$
3. Prove that if a positive integer is divisible by 8 then it is the difference of two perfect squares. Is your proof direct, by contradiction, or by contrapositive? Is it constructive or nonconstructive?
4. Prove that the equation $x^{101} + x^{51} + x + 1 = 0$ has exactly one real solution. Split this into two statements:
   (a) the equation has at least one solution. Is your proof constructive or nonconstructive?
   (b) the equation can not have two distinct roots. Is your proof direct, by contradiction, or by contrapositive?
5. Every odd number is either of the form $4n + 1$ (if it has remainder 1 when divided by 4) or of the form $4n+3$ (if it has remainder 3). Prove that if an odd number is a perfect square, then it has the form $4n + 1$. Is your proof direct, by contradiction, or by contrapositive? State the converse. Prove or disprove the converse.

For extra credit: Prove or disprove that if a and b are irrational numbers, then a^b is also irrational.