# Homework 4

## Number theory

**Due 26 September 2003**, 5 points each:

1. Show that $\sqrt[3]{25}$ is irrational.
2. If $c$ is a perfect square, what are the possible values of its last (units) digit? Conclude that a number ending with 3 cannot be a perfect square.
3. Show that 3 divides both $a$ and $b$ iff 3 divides $a^2 + b^2$.
4. Recall the following problem done in class: "show that a natural number is divisible by 3 iff the sum of its digits is divisible by 3". Show similarly that a natural number is divisible by 9 iff the sum of its digits is divisible by 9. Derive that if the sum of the digits of a number is 66, then it is not a perfect square.
5. Show that if $n$ is not prime, then $2^n - 1$ is not prime.

**Extra credit:** Find all integral solutions of $x + y = x^2 - xy + y^2$.