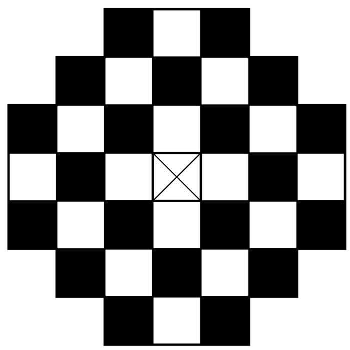
Math 145 Fall 2003

Homework 7 - Solutions

Coloring

Due 20 October 2003, 5 points each:

1. **Prove that the figure shown below (with center block removed) cannot be covered by dominoes.**



   There are 36 squares, and each domino covers 2, so we need 18 dominoes. Color the figure as a chessboard. It has 20 black and 16 white squares. Since each domino covers one black and one white square, 18 dominoes must cover 18 black and 18 white squares while we have 20 and 16. So it is not possible to cover the figure with dominoes.

2. **Prove that a 10×10 chessboard cannot be covered by 25 T-tetrominoes.**

   A 10×10 chessboard has 50 black and 50 white squares. Each T-tetromino covers either 3 black and 1 white or 1 black and 3 white squares. Suppose there are a T-tetrominoes covering 3 black squares. Then there are $25 - a$ T-tetrominoes covering 1 black square. Then all 25 tetrominoes cover $3a + (a - 25) = 2a - 25$ black squares. They must cover 50, so $2a - 25 = 50$, or $2a = 25$. This equation has no integer solutions since 25 is not divisible by 2.

3. **The vertices and midpoints of the faces are marked on a cube, and all face diagonals are drawn. Prove that there is no path along the face diagonals that visits each marked point exactly once.**

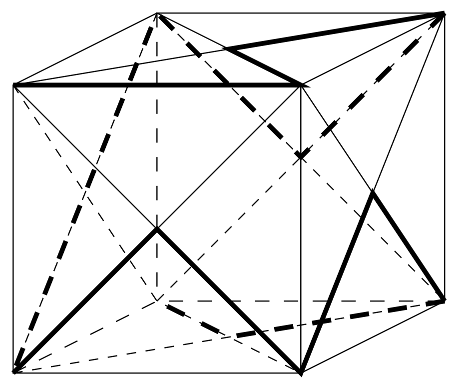
   Notice that every piece of a face diagonal connects a vertex and a face midpoint. Thus if we only use face diagonals, vertices and midpoints must alternate. But there are 8 vertices and 6 midpoints, so there is no way to make them alternate (there are too many vertices).

(Note: we could color all the marked points: let vertices be black, and let midpoints be white... then black and white points must alternate, but there are 8 black points and 6 white points, so that's impossible.)

4. **Show that if in the previous problem one walk along an edge is allowed, then there is a path visiting all the marked points. (Find such a path.)**

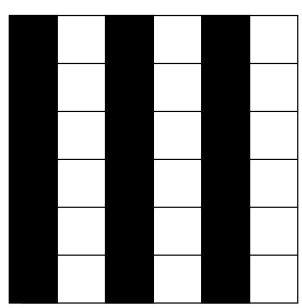
   If one edge is allowed, then we could have two vertices in the beginning, after which we would be left with 6 midpoints and 6 vertices, and we can make them alternate. Again, let vertices be black and midpoints white, then a path could be e.g. $bbwbwbwbwbwbwb$.

   Here is an example. (But there are many other such paths.)



5. **Prove that a 6×6 board cannot be covered by 9 L-tetrominoes.**

   Cover the board as shown below.



   There are 18 black squares. The rest of the argument is the same as in problem 2. Each L-tetromino covers either 1 or 3 black squares. Let a tetrominoes cover 3 black squares, then $9 - a$ tetrominoes cover 1 black square, and all 9 together cover $3a + (9 - a) = 2a + 9$. Therefore $2a + 9 = 18$, or $2a = 9$, but this equation has no integer solutions.