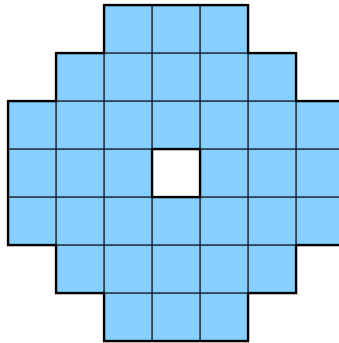


## Homework 7

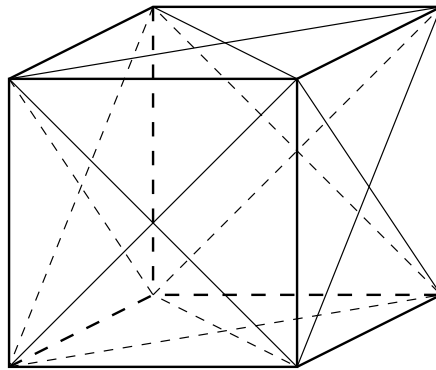
### Coloring

Due 20 October 2003, 5 points each:

1. Prove that the figure shown below (with center block removed) cannot be covered by dominoes.



2. Prove that a  $10 \times 10$  chessboard cannot be covered by 25 T-tetrominoes.
3. The vertices and midpoints of the faces are marked on a cube, and all face diagonals are drawn. Prove that there is no path along the face diagonals that visits each marked point exactly once.



4. Show that if in the previous problem one walk along an edge is allowed, then there is a path visiting all the marked points. (Find such a path.)
5. Prove that a  $6 \times 6$  board cannot be covered by 9 L-tetrominoes.

**Extra credit:** The map below shows the cities and one-way roads in Sikinia. Prove that there is no closed path (a path is closed if it starts and quits in the same city) that visits every city exactly once. Is there a path that visits every city exactly twice? How about a path, not necessarily closed, that starts in the upper left corner and visits every city exactly once? Finally, is there a path, not necessarily closed, that starts in the upper left corner and visits every city exactly twice?

