

Homework 9

Graphs

Due 3 November 2003:

1. (2 pts) Explain why a graph can not have 7 vertices of degrees 4, 4, 3, 3, 3, 2, 2.
2. (5 pts) Prove that in any group of people, the number of people that are friends with an odd number of people is even.
3. (8 pts) For what values of n and m does $K_{n,m}$ have
 - (a) an Euler cycle?
 - (b) an Euler path?
 - (c) a Hamilton cycle?
 - (d) a Hamilton path?
4. (5 pts) Show that there is no reentrant knight's tour on a 4×4 chessboard. Actually, there is no tour at all, but that is a bit harder to prove.
5. (5 pts) There are 7 men and 7 women attending a dance. After the dance, they recall the number of people they have danced with. The numbers are as follows: 3, 3, 3, 3, 3, 3, 5, 6, 6, 6, 6, 6. Prove that at least one of them made a mistake. (Assume that men only danced with women, and women only danced with men.)

Extra credit: Show that there is no knight's tour at all (reentrant or not) on a 4×4 chessboard.