

Number theory

Problems

1. Show that $\log_2 5$ is irrational.
2. If c is a perfect square (the square of an integer), what are the possible values of its remainder upon division by 4? Conclude that a number ending with 62 cannot be a perfect square.
3. Can a number ending with 65 be a perfect square?
4. Show that for any natural n , $6|n^3 + 5n$.
5. Show that a natural number is divisible by 3 iff the sum of its digits is divisible by 3.
6. Find $2^{100} \bmod 5$ (that is, find the remainder upon division of 2^{100} by 5).
7. Show that $A = 3^{105} + 4^{105}$ is divisible by 7. Find $A \bmod 11$ and $A \bmod 13$.
8. Find all primes p and q such that $p^2 - 2q^2 = 1$.
9. Find all the integral solutions of $x + y = xy$.
10. Show that $x^2 - 3y^2 = 17$ has no integral solutions.
11. Show that $2^n \nmid n!$.
12. The four-digit number $aabb$ is a perfect square. Find it.