

Finding a pattern

Problems

- Find a formula for the n -th term of the sequence $\{a_1, a_2, a_3, \dots\}$ whose first few terms are given.
 - 1, 0, 1, 2, 3, 4, 5, ...
 - 5, 7, 9, 11, 13, 15, ...
 - 1, 3, 4, 6, 7, 9, 10, 12, ...
 - $\frac{1}{2}, \frac{1}{2}, \frac{3}{8}, \frac{1}{4}, \frac{5}{32}, \frac{3}{32}, \frac{7}{128}, \dots$
- Find the n -th derivative of
 - $f(x) = \ln(x)$
 - $g(x) = \sin(x)$
- n lines in general position are given in a plane. (General position means that no two lines are parallel, and no 3 lines have a common point.) Into how many regions do they divide the plane?
- What is the last digit of 107^{107} ? (Hint: find the last digit of 103^n for small values of n and notice the pattern.)
- Let $f_1(x) = \frac{1}{2-x}$ and $f_{n+1} = f_1 \circ f_n$ for $n \geq 1$. Find an expression for $f_n(x)$ and use Mathematical Induction to prove it.
- Amanda is training her rabbit to climb a flight of 10 steps. The rabbit can hop up 1 or 2 steps each time he hops. He never hops down, only up. How many different ways can he hop up the flight of 10 steps? (Don't even try to list all the ways. There are too many of them! Better replace 10 by small numbers, and guess the pattern. Prove your guess using Mathematical Induction.)