Practice Test 1

Answer the question (5 points):

- What does "a and b are relatively prime" mean?

and do any 3 of the following problems (15 points each):

1. Prove that if n is an integer then $n^2 + 2$ is not divisible by 5.

2. Prove that for any natural n,

$$1 \cdot 1! + 2 \cdot 2! + \ldots + n \cdot n! = (n+1)! - 1.$$

3. Let $P(x, y)$ denote the proposition "$x < y$" where x and y are real numbers. Determine the truth values of

    (a) $\exists x \exists y P(x, y)$,

    (b) $\forall x \exists y P(x, y)$,

    (c) $\exists x \forall y P(x, y)$,

    (d) $\forall x \forall y P(x, y)$.

4. Let $f_1(x) = 2x + 1$ and $f_n = f_1 \circ f_{n-1}$. Compute f_n for some small values of n. Notice the pattern. Write a formula for f_n and prove it using Mathematical Induction.

Extra credit (15 points):

- Six points are selected inside a 3×4 rectangle. Prove that there two of them such that the distance between them is at most $\sqrt{5}$.