

Practice Test 2

Answer the question (5 points):

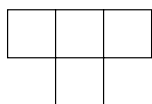
- What is an Euler cycle?

and do any 3 of the following problems (15 points each):

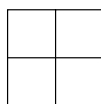
1. Start with the set $\{1, 1, 1, 1\}$. In each step, you may either multiply one of the numbers by 3, or subtract 2 from it. Show that it is not possible to reach the set $\{1, 2, 3, 4\}$.
2. An 8×8 chessboard is covered by tetrominoes (all 5 kinds are shown below). Prove that the number of T-tetrominoes is even.



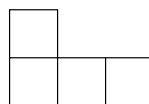
straight
tetromino



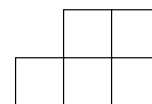
T-tetromino



square
tetromino



L-tetromino



skew
tetromino

3. Solve for x : $|x - 5| + |2x - 4| \leq 6$.
4. There are 8 counties in Sikinia. There are no “four corners” points (like Arizona, Colorado, New Mexico, and Utah). Each county counted the number of neighboring counties. The numbers are 5, 5, 4, 4, 4, 4, 4, 3. Prove that at least one county made a mistake.

Extra credit (15 points):

- Nine 1×1 cells of a 10×10 square are infected. In one time unit, the cells with at least two infected neighbors (having a common side) become infected. Can the infection spread to the whole square?