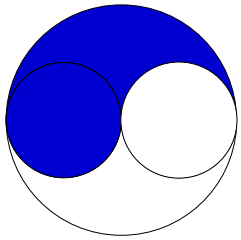
Symmetry, Translations, Rotations, and Similarity

Problems

In all the problems below, "find" means "construct", or "draw". You do not have to calculate the locations of all the points. Assume that solutions exist.

1. (a) Two points A and C, and a line are given. Find a point B on the line such that $AB + BC$ is a minimum.
   (b) A circle, a line, and a point C are given. Find a point A on the circle and a point B on the line such that $AB + BC$ is a minimum.
2. Three lines are given. Find three points on these lines, one point on each line, that are vertices of an equilateral triangle. (Hint: you can choose any point on the first line as one of the vertices.)
3. Show that among all rectangles with given perimeter, the square has the maximal area.
4. A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?
5. Two circles are given. Draw a line that is tangent to both circles and such that both circles lie on one side of the line.
6. The shaded region in the figure is bounded by three semi-circles. Cut this region into four congruent parts, i.e. parts of equal size and shape.



7. Two lines, p and q, and a point A are given. Draw a square $ABCD$ that satisfies the following conditions:
   - The vertex A is the given point.
   - The vertex B, the counterclockwise neighbor-vertex of A, lies on the line p.
   - The vertex C, the counterclockwise neighbor-vertex of B, lies on the line q.