MATH 145

Test 1

26 September 2003

Name: ______________________________

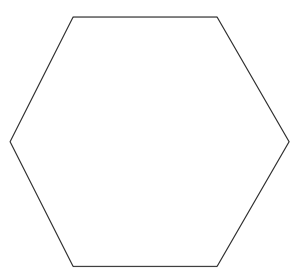
Answer the question (5 points):

- Let $P(x, y)$ be a propositional function. Are $\forall x \exists y P(x, y)$ and $\exists y \forall x P(x, y)$ logically equivalent?

  Answer ("yes" or "no"): __________

and do any 3 of the following problems (15 points each):

1. Prove that among 120 integers, there are two whose difference ends with 00.
2. Compute $A_n = 1 + 3 + 5 + \ldots + (2n - 1)$ for some small values of n. Notice the pattern. Write a formula for A_n and prove it using Mathematical Induction.
3. Prove that for every integer n, $n^3 + 2n$ is divisible by 3.
4. 7 points are selected inside a regular hexagon whose sides have length 1. Prove that there are two points such that the distance between them is at most 1.



Extra credit (15 points):

- Prove that among $n + 1$ positive integers all less than or equal to $2n$, there are two which are relatively prime.

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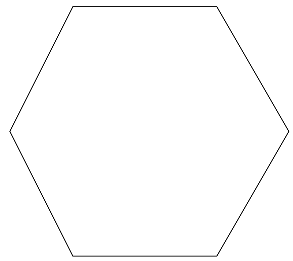

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