MATH 145

Test 1

26 September 2003

Name: _________________________________

Answer the question (5 points):

• Let \( P(x, y) \) be a propositional function. Are \( \forall x \exists y P(x, y) \) and \( \exists y \forall x P(x, y) \) logically equivalent?

Answer ("yes" or "no"): _______

and do any 3 of the following problems (15 points each):

1. Prove that among 120 integers, there are two whose difference ends with 00.
2. Compute \( A_n = 1 + 3 + 5 + \ldots + (2n - 1) \) for some small values of \( n \). Notice the pattern. Write a formula for \( A_n \) and prove it using Mathematical Induction.
3. Prove that for every integer \( n \), \( n^3 + 2n \) is divisible by 3.
4. 7 points are selected inside a regular hexagon whose sides have length 1. Prove that there are two points such that the distance between them is at most 1.

Extra credit (15 points):

• Prove that among \( n + 1 \) positive integers all less than or equal to \( 2n \), there are two which are relatively prime.
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