MATH 145

Test 2

7 November 2003

Name: _____

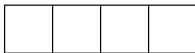**Answer the question** (5 points):

- What is a Hamilton path?
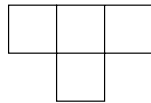
  Answer: _____

  _____

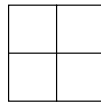**and do any 3 of the following problems** (15 points each):

1. Start with the set $\{-3, -2, -1, 1, 2, 3\}$. In each step you may choose any two of these numbers and change their signs. Show that it is not possible to reach the set $\{3, 2, 1, 1, 2, 3\}$.

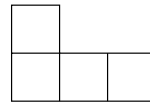2. Prove that an 8×8 square cannot be covered by 11 straight tetrominoes and 5 L-tetrominoes.
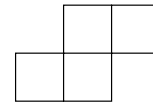

straight tetromino   T–tetromino   square tetromino   L–tetromino   skew tetromino
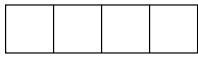
3. Solve for x: $|2x + 3| - |x| = 3$.

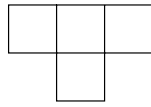4. Show that there is no reentrant knight's tour on a 5×5 chessboard.

- **Extra credit** (15 points): Twelve 1×1 cells of a 10×10 square are infected. Two cells are neighbors if they share at least one vertex (thus an inner cell has 8 neighbors). In one time unit, the cells with at least 4 infected neighbors become infected. Can the infection spread to the whole square?

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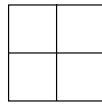
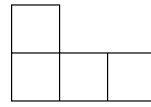2. Prove that an 8×8 square cannot be covered by 11 straight tetrominoes and 5 L-tetrominoes.
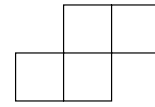
straight
tetromino

T−tetromino

square
tetromino

L−tetromino

skew
tetromino

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Extra credit: Twelve 1×1 cells of a 10×10 square are infected. Two cells are neighbors if they share at least one vertex (thus an inner cell has 8 neighbors). In one time unit, the cells with at least 4 infected neighbors become infected. Can the infection spread to the whole square?