MATH 145

Test 1

3 October 2005

Name: ____

Answer the question (5 points):

- If $P(x)$ is a propositional function, which of the following are logically equivalent: $\neg\exists x P(x)$, $\exists x \neg P(x)$, $\forall x \neg P(x)$?

  Answer: ____

and do any 3 of the following problems (15 points each):

1. Let $P(x,y)$ denote the propositional function "$xy = 0$" where x and y are real numbers. Determine the truth values of the following propositions. (Provide reasons!)
   (a) $\exists x \forall y P(x,y)$,
   (b) $\forall x \exists y P(x,y)$,
   (c) $\forall x \forall y P(x,y)$,
   (d) $\exists! x \forall y P(x,y)$,
   (e) $\forall x \exists! y P(x,y)$.
2. Prove that if a is rational and b is irrational then $a+b$ is irrational.
   Is your proof direct, by contradiction, or by contrapositive?
3. Use Mathematical Induction to prove that for any natural n,
   $$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}.$$
4. Ten points are chosen randomly inside a 3×3 square. Prove that there are two of them with distance at most $\sqrt{2}$.

For extra credit (15 points):

- Prove that for any integer number $n \geq 3$, $\left(1 + \frac{1}{n}\right)^n < n$.

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   (b) $\forall x \exists y P(x, y)$

   (c) $\forall x \forall y P(x, y)$

   (d) $\exists! x \forall y P(x, y)$

   (e) $\forall x \exists! y P(x, y)$

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