MATH 145

Test 2

4 November 2005

Name: _____

Answer the question (5 points):

- Is it true or false that if a and b are not integers then $\frac{a}{b}$ is irrational?

  Answer: _____

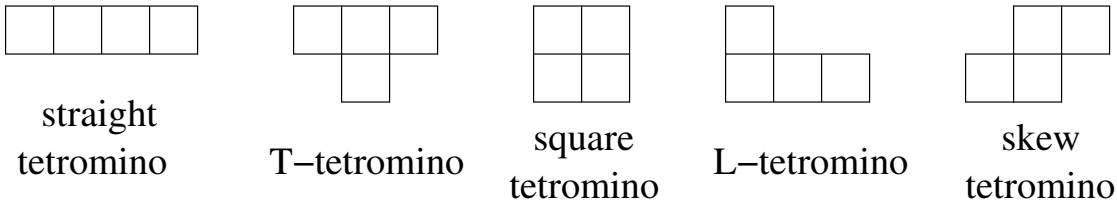
and do any 3 of the following problems (15 points each):

1. Solve for x: $(x-3)^{x^2-8x+15} = 1$
2. Find a formula for

$$\prod_{i=1}^{2n-1}\left(1-\frac{(-1)^i}{i}\right) = \left(1-\frac{-1}{1}\right)\left(1-\frac{1}{2}\right)\left(1-\frac{-1}{3}\right)\cdots\left(1-\frac{-1}{2n-1}\right)$$

   and prove it.

3. Start with the set $\{1, 4, 32, 128, 256\}$. In each step, you may divide one number by 2 and multiply another number by 2. Is it possible to reach the set $\{512, 32, 16, 16, 2\}$?
4. Prove that an 8×8 board cannot be covered by 7 T-tetrominoes and 9 L-tetrominoes.



- **Extra credit** (15 points): Which natural numbers are sums of consecutive smaller natural numbers? For example, $30 = 9 + 10 + 11$ and $31 = 15 + 16$, but 32 has no such representation. Find a simple condition and prove it.

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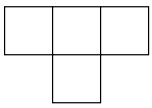
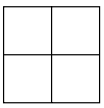
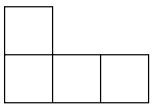
straight tetromino



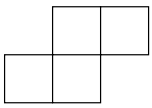
T–tetromino



square tetromino



L–tetromino



skew tetromino

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