MATH 145

Test 3

5 December 2005

Name: ______________________________

Answer the question: is the following statement true of false? (5 points):

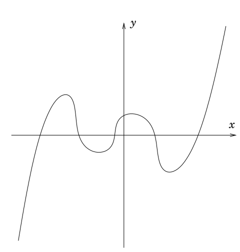
- If $\int_a^b f(x)dx > 0$ then $f(x) \geq 0$ for all $x \in [a, b]$.

  Answer: ____________

and do any 3 of the following problems (15 points each):

1. Sketch the region $S = \{(x, y) \mid x \geq 2, x^2 + y^2 \leq 16\}$ and find its area.
2. A graph $K_{k,l,m}$ has $k + l + m$ vertices divided into three sets: k vertices in one set, l vertices in another set, and m vertices in the third set. Two vertices are connected if and only if they are in different sets. Does $K_{1,2,4}$ have an Euler cycle?
3. Two players play the following game.
   - Turns alternate.
   - At each turn, a player removes 1, 2, or 4 counters from a pile that had initially 10 counters.
   - The game ends when all counters have been removed.
   - The player who takes the last counter wins.

   Find a winning strategy for one of the players.
4. Explain why the curve shown below cannot be the graph of a cubic polynomial.



- **Extra credit** (15 points): What is the ratio of the 5-dimensional volume of a 5-dimensional ball to the 4-dimensional volume of its boundary (the analog of the surface area)?

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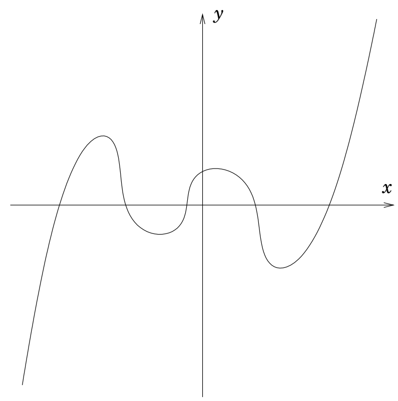

For extra credit: What is the ratio of the 5-dimensional volume of a 5-dimensional ball to the 4-dimensional volume of its boundary (the analog of the surface area)?