THE PLAYGROUND!

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Welcome to the Playground! Playground rules are posted at the end of page 33, except for the most important one: *Have fun*!

THE SANDBOX

In this section, we highlight problems that anyone can play with, regardless of mathematical background. But just because these problems are easy to approach doesn't necessarily mean that they are easy to solve!

The points on the real number line $\boldsymbol{R}$ have the special property of being *totally ordered*: any two real numbers a and b are related either by $a \leq b$ or $b \leq a$. (Many thanks to the 1-dimensional line for making this possible!) For the Sandbox problem, we consider a slightly different type of order, called a *partial order*, for points in 2-dimensional and 3-dimensional space since these points don't lie naturally on a 1-dimensional line.

For two points (a,b) and (c,d) in the 2-dimensional plane $\boldsymbol{R}^2$, we'll say that $(a,b) \leq (c,d)$ if both $a \leq c$ and $b \leq d$ are true for the real numbers a, b, c, and d. For example, $(1,5) \leq (3,11)$ and $(-1,-2) \leq (5,-2)$; but it is not true that $(1,\pi) \leq (203,3)$ because it is not true that $\pi \leq 3$. Nor is it true that $(203,3) \leq (1, \pi)$, which means that only some pairs of points in $\boldsymbol{R}^2$ are related by this new notion of "less than or equal to."

In an analogous "coordinate-wise" way we can define $\leq$ for points in $\boldsymbol{R}^3$: we'll say that $(a,b,c) \leq (d,e,f)$ if $a \leq d$, $b \leq e$, and $c \leq f$.

As a way to first start thinking about this new type of order, try to find three points in $\boldsymbol{R}^2$ such that no pair of the points is related by $\leq$. An example of such points is given later in the section "Cleaning Up."

Problem 232, Orderly Points, is offered by Gary Gordon of Lafayette College. Suppose that you are given a set of infinitely-many distinct points (x,y) in $\boldsymbol{R}^2$, where all of the x and y values are positive integers.

(a) Show that there must be a pair of distinct points (x_1, y_1) and (x_2, y_2) such that $(x_1, y_1) \leq (x_2, y_2)$.

(b) Show that in fact there must be a chain of infinitely-many such distinct points:

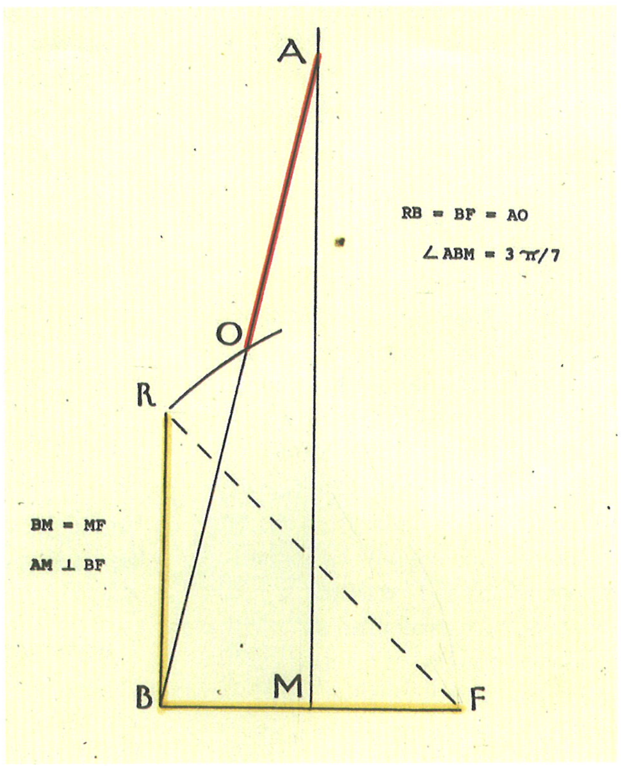
$$(x_1, y_1) \leq (x_2, y_2) \leq (x_3, y_3) \leq (x_4, y_4) \leq \ldots$$

(c) What happens in $\boldsymbol{R}^3$? If you are given a set of infinitely-many distinct points (x,y,z) in $\boldsymbol{R}^3$, where x, y, and z are positive integers, must there be two distinct points related by $\leq$? An infinite chain of them?

THE ZIP-LINE

This section offers problems with connections to articles that appear in this issue. Not all of the problems in this section require you to read the corresponding articles, but doing so can never hurt, of course.

The following figure is copied from the article "Harold and the Purple Heptagon" on pages 5-9. Line segments RB and BF form two sides of a square, and the curve drawn through points O and R is an arc of a circle centered at F. AM is a perpendicular bisector of BF, and AO has the same length as BF. **Problem 233, A Slice of Purple Pi,** is to show that $\angle ABF = (3/7)\pi$.



One way to begin thinking about the problem is to ask yourself how you might prove, in any situation, that a given angle has measure of the form $(k/7)\pi$ for an integer k.