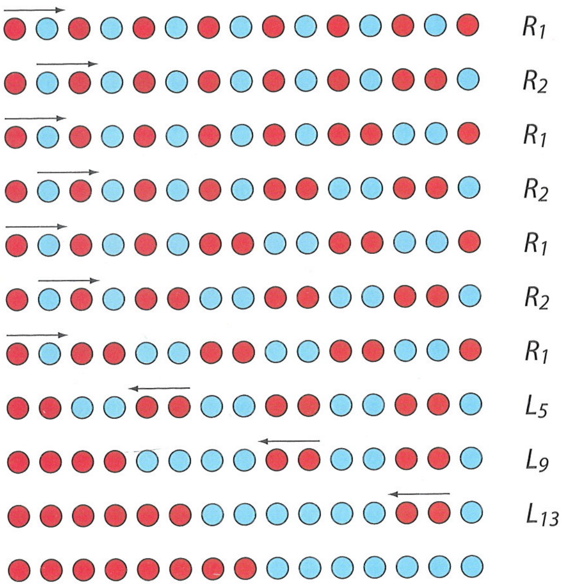



The first six solvers listed above for (a) also submitted correct solutions for part (b), each describing a specific sequence of moves for sorting the chips that breaks into two cases, depending on whether $n$ is even or odd. All of the solutions presented were variants of the following method, modeled on the solution from the group from Armstrong Atlantic and generalizing the example presented above in part (a). Repeat the moves $R_1$ and $R_2$ a total of $\lfloor (n-1)/2 \rfloor$ times, followed by one more $R_1$ move (where $\lfloor x \rfloor$ is the "floor" of $x$, the greatest integer less than or equal to $x$). At this point, all of the red chips will be paired together, with the exception of one red chip on the left end of the row in the case when $n$ is even. Regardless of the parity of $n$, you can check that then $\lfloor n/2 \rfloor$ pairs of red chips may be moved to the left end of the row to complete the sort, for a grand total of

$$2\lfloor (n-1)/2 \rfloor + 1 + \lfloor n/2 \rfloor$$

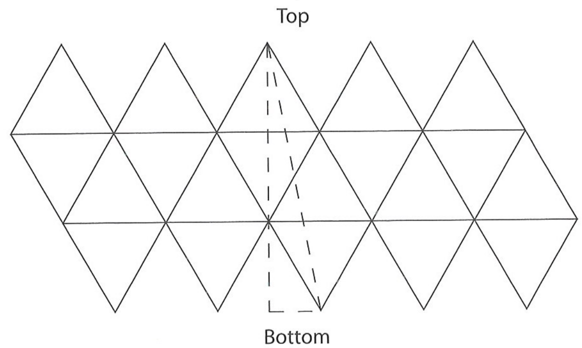
moves. You can also check that this last expression is in fact equivalent to $\lfloor (3n-1)/2 \rfloor$ by considering its value when $n$ is either even or odd.

This shows that $s(n)$ is bounded above by $g(n) = \lfloor (3n-1)/2 \rfloor$ for all $n$, answering the question in part (b). For part (c), several submissions attempted to show that $s(n) = g(n)$ by explaining why any possible sorting method would require at least $g(n)$ moves. None of the proposed solutions were completely successful, so we'll keep this part of the problem open.

**An Icosahedral Climb** described the situation of an adventurous ant approaching an icosahedron that is balancing on one of its vertices. **Problem 225** asked you to help the ant find the path of shortest distance to the top of the icosahedron.

One obvious way to the top follows a zig-zag-zig path along 3 of the 30 edges of the icosahedron. If the length of one edge is $s$, then the total length of this path is $3s$. A more direct path first sends the ant up an altitude of one triangle, then up a second altitude, and finally along an edge to the top, a path that has a total length of $(1+\sqrt{3})s$.

However, the ant can do better, as described in solutions from students Tim Hayes and Michael Abram, and the Armstrong Problem Solvers, the Bethel College Problem-Solving Group, and Taylor A. The key is to "unfold" the icosahedron, such as in the figure below, and then determine the shortest Euclidean distance from any point corresponding to the bottom vertex to any point corresponding to the top vertex.




Tim explains how the length of the slanted dotted path shown above can be computed using the Pythagorean Theorem: since the short dotted leg has length $s/2$ and the long dotted leg has the length of three triangle altitudes, the total path length is

$$\sqrt{(s/2)^2 + (3\sqrt{3}s/2)^2} = s\sqrt{7}.$$

But we were asked to help the ant, not just compute the length of the shortest path! The Armstrong Problem Solvers give the ant explicit instructions:

> From the bottom vertex, crawl in a straight line along any of the lower five faces to a point 1/3 of the way along the opposite edge of that face. On the next face, crawl in a straight line to the midpoint of the nearer of the other two edges in that face. Crawl up the next face to a point 1/3 of the way along the horizontal edge of that face. Finally, crawl on the next face to the opposite vertex, which will be the top.

The group from Bethel College challenges you to solve the analogous problem for the cube, octahedron, and dodecahedron. (For an action-packed image of someone considering the problem on the cube, look no further than page 21 of last November's *Horizons*.) For another challenge of this type, find one of the many online references to Henry Dudeney's classic problem of the spider and the fly.