Math 145 **Fall 2009**

Practice Test 1

Note: On the actual test, there will be 4 problems, and you will be asked to choose any 3 problems.

1. Let $P(x, y)$ denote the proposition "$x \leq y$" where x and y are real numbers. Determine the truth values of
   (a) $\forall x P(-x, x)$,
   (b) $\exists x \exists y P(x, y)$,
   (c) $\forall x \exists y P(x, y)$,
   (d) $\exists x \forall y P(x, y)$,
   (e) $\forall x \forall y P(x, y)$.
2. Prove that for any integers n and m, if $nm + 2n + 2m$ is odd, then both n and m are odd (you may only use the definitions of even and odd numbers; do not use any properties unless you prove them). Is your proof direct, by contrapositive, or by contradiction?
3. Prove or disprove that the product of a rational number and an irrational number is always irrational.
4. Prove that for any natural n,
$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$
5. Let $\{F_0,\ F_1,\ F_2,\ \ldots\}$ be the Fibonacci sequence defined by $F_0 = 0$, $F_1 = 1$, and $F_{n+1} = F_n + F_{n-1}$, $n \geq 1$. Prove that $F_{n-1}^2 + F_n^2 = F_{2n-1}$
6. Kevin is paid every other week on Friday. Show that every year, in some month he is paid three times.
7. Let f be a one-to-one function from $X = \{1, 2, 3, 4, 5\}$ onto X. Let $f^k = \underbrace{f \circ f \circ \ldots \circ f}_{k \text{ times}}$ denote the k-fold composition of f with itself. Show that for some positive integer m, $f^m(x) = x$ for all $x \in X$.
8. Prove that the sum of four consecutive odd integers cannot end with 7.
9. How many pairs of positive integers are solutions to the equation $2x + 3y = 100$?