Practice Test 2

Answer the question (5 points):

- Is it true or false that an integer n is divisible by 12 if and only if it is divisible by both 2 and 6?

and do any 3 of the following problems (15 points each):

1. Show that $2^{457} + 3^{457}$ is divisible by 5.

2. Solve for x: $|x + 1| + 5 - x^2 \geq 0$

3. Let $F_0 = 0$, $F_1 = 1$, $F_2 = 1$, ..., F_{99} be the first 100 Fibonacci numbers (recall that $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$). How many of them are even?

4. There are seven 1's and eight -1's on a blackboard. In each step, you may erase any two numbers, say, a and b, and write $-ab$ instead. Show that no matter in what order we erase the numbers, 1 will remain in the end.

Extra credit (15 points):

- Show that if $4 \times 1 \times 1$ bricks and $2 \times 2 \times 2$ cubes fill (without overlap) an $8 \times 8 \times 8$ cube, then the number of $2 \times 2 \times 2$ cubes is even.