# MATH 145

# Test 1

September 30, 2009

**This test is to be taken on a furlough day.**
**It is take-home, self-check, and not part of your grade.**

- No books, notes, or calculators are allowed.
- Please show all your work. Prove all your claims.
- Choose any three problems.

1. Prove that $\sqrt[3]{5}$ is irrational. Is your proof direct, by contradiction, or by contrapositive?

2. Let $\{F_0,\ F_1,\ F_2,\ \ldots\}$ be the Fibonacci sequence defined by $F_0 = 0$, $F_1 = 1$, and $F_{n+1} = F_n + F_{n-1}$, $n \geq 1$. Prove that

$$F_0 - F_1 + F_2 - F_4 + \ldots - F_{2n-1} + F_{2n} = F_{2n-1} - 1.$$

3. Suppose three numbers, $a_1$, $a_2$, and $a_3$, are randomly chosen from the set

$$\{2, 4, 5, 8, 16, 25, 32, 64, 125, 128\}.$$

   Prove that at least one of the six quotients $a_i/a_j$, for $i \neq j$, is an integer.

4. Find the last digit of the number $2^{2009} + 3^{2009}$.

- **For extra credit:** Consider the sequence

$$0,\ 1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,\ 55,\ 89,\ 44,\ 33,\ 77,\ 10,\ 87\ldots$$

  This sequence starts the same way as the Fibonacci sequence, and each term starting with the third is the sum of the two preceding terms, but addition is done modulo 100. Prove that this sequence is periodic.