MATH 145

Test 3

December 4, 2009

This test is to be taken on a furlough day.
It is take-home, self-check, and not part of your grade.

- No books, notes, or calculators are allowed.
- Please show all your work. Prove all your claims.
- Choose any three problems.

1. Prove that if all four corner squares are removed from an 8×8 board, then the obtained board cannot be covered with T-tetrominoes.

2. The sequence $1, 2, 3, 6, 11, 8, \ldots$ satisfies $a_n \cong a_{n-1} + a_{n-2} + a_{n-3} \pmod{12}$. Prove that the triple $3, 6, 9$ will never occur.

3. In one family there are six children, two sets of triplets with age difference 2 years. All six attend the same university. Each of them counted the number of siblings that they take classes with this semester. These numbers are: 1, 1, 1, 2, 3, 3. Prove that at least one of them made a mistake.

4. Find the equations of both tangent lines to the circle $x^2 + y^2 = 1$ that pass through the point $(2, 2)$.

- **For extra credit:** Find the area of the intersection of two ellipses: $(x-1)^2 + 4y^2 = 4$ and $(x+1)^2 + 4y^2 = 4$.