## Practice Test 3

Do any 3 of the following problems:

1. Prove that a $6 \times 6$ board cannot be covered by L-tetrominoes.
2. A circle of radius 2 passes through the center of a circle of radius 1 (see picture below). Find the area of the shaded triangle.

3. A graph $K_{k, l, m}$ has $k+l+m$ vertices divided into three sets: $k$ vertices in one set, $l$ vertices in another set, and $m$ vertices in the third set. Two vertices are connected if and only if they are in different sets. Prove that $K_{1,3,5}$ has a Hamilton path but not a Hamilton cycle.
4. Two players play the following game. Turns alternate. At each turn, a player removes 1,2 , or 4 coins from a pile that initially had 10 coins. The game ends when all coins have been removed. The player who cannot make a move loses. Find a winning strategy for one of the players.

## Extra credit:

- Is it possible for a chess knight to pass through all the squares of a $4 \times$ 2005 board having visited each square exactly once, and return to the initial square?

