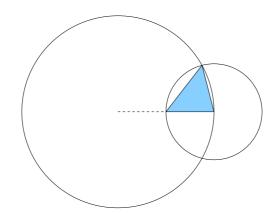
Fall 2013

Practice Test 3

Do any 3 of the following problems:

- 1. Prove that a 6×6 board cannot be covered by L-tetrominoes.
- 2. A circle of radius 2 passes through the center of a circle of radius 1 (see picture below). Find the area of the shaded triangle.



- 3. A graph $K_{k,l,m}$ has k+l+m vertices divided into three sets: k vertices in one set, l vertices in another set, and m vertices in the third set. Two vertices are connected if and only if they are in different sets. Prove that $K_{1,3,5}$ has a Hamilton path but not a Hamilton cycle.
- 4. Two players play the following game. Turns alternate. At each turn, a player removes 1, 2, or 4 coins from a pile that initially had 10 coins. The game ends when all coins have been removed. The player who cannot make a move loses. Find a winning strategy for one of the players.

Extra credit:

• Is it possible for a chess knight to pass through all the squares of a 4 × 2005 board having visited each square exactly once, and return to the initial square?