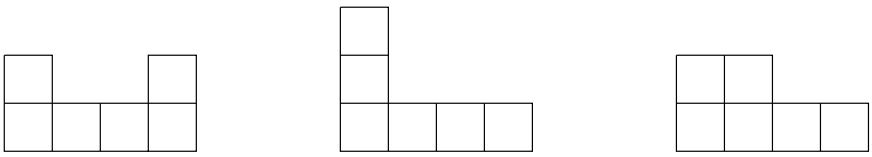
Math 145 **Fall 2014**

Review Problems

1. Find $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots + \frac{n-1}{n!}$.
2. Prove that if $p > 3$ is prime, then $p^2 \equiv 1 \pmod{24}$.
3. There are 8 people in a room. Every person counted how many people he knows. (Assume that if A knows B then B knows A.)
   (a) The numbers are 0, 1, 1, 2, 2, 3, 4, 4. Prove that somebody made a mistake.
   (b) Can the numbers be 0, 1, 2, 3, 4, 5, 6, 7?
4. We strike the first digit of the number 7^{2003}, and add it to the remaining number. This is repeated until a number with 10 digits remains. Prove that this number has 2 equal digit.
5. Show that it is not possible to cover any rectangle by one tile of type 1 shown below, one tile of type 3, and any number of tiles of type 2.



1 2 3

6. Let S be a set of 25 points such that, in any 3-subset of S, there are at least 2 points with distance less than 1. Prove that there exists a 13-subset of S which can be covered by a disk of radius 1.