

MATH 145

Test 1

October 1, 2014

Name: _____

- No books, notes, or calculators are allowed.
 - Please show all your work. Prove all your claims.
 - Choose any three problems from problems 1-4. Please circle or clearly mark the three problems you want to be graded. Each of these problems is worth 20 points.
 - The extra credit problem is worth 10 points.
1. Let $P(x, y)$ denote the open statement (a.k.a. propositional function) “ $x - y = 2014$ ” where x and y are real numbers. Determine the truth value of the following statements:
 - (a) $\forall x \exists y P(x, y)$
 - (b) $\exists y \forall x P(x, y)$
 - (c) $\exists x \exists ! y P(x, y)$
 - (d) $\exists ! y \exists x P(x, y)$
 2. Let n be an integer. Prove that if $145n - 1$ is odd, then $n + 2014$ is even. Is your proof direct, by contrapositive, by contradiction, or none of these? (If it is none of the listed proof types, then describe/summarize your proof type/strategy.)
 3. Prove that any integer $n \geq 4$ can be written as $n = 2a + 5b$ for some non-negative integers a and b .
 4. Ten points are chosen randomly in a 6×6 square. Prove that at least two of these points are within distance 3 of each other.
- **For extra credit:** Assume in a class of students each of a number of committees contains more than half of all the students. Prove that there is a student who is a member in more than half of the committees. Then prove that if the number of committees is 30, then it is possible to select just 4 students to be committee representatives so as not to leave a committee without a representative. (Committees may be only represented by their own members, but one student may represent any number of committees.)