MATH 145

Test 2

October 31, 2014

Name: _____

- No books, notes, or calculators are allowed.

- Please show all your work. Prove all your claims.

- Choose any three problems. Please make your choice clear.

1. Prove that $1^{2014} + 2^{2014} + 3^{2014}$ is divisible by 7.

2. Solve over $\mathbb{R}$: $x^2 + 3 \leq |3x - 3| + 4x$.

3. Let $F_0 = 0$, $F_1 = 1$, ..., F_{2013} be the first 2014 Fibonacci numbers. How many of them are divisible by 4?

4. We start with the set $\{1, 2, 3, 4, 5, 6\}$. In each step we may either multiply any one of these numbers by 3 or add 2 to it. We may repeat this step as many times as we want. Prove that it is impossible to reach the set $\{18, 27, 36, 54, 81, 108\}$.

- **For extra credit:** Do there exist integers m and n such that $m^2 + 20142015 = n^2$?