MATH 145

Test 3

December 5, 2014

Name: _____

- No books, notes, or calculators are allowed.

- Please show all your work. Prove all your claims.

- Choose any three problems.

1. Prove that a 10×10 board cannot be covered with T-tetrominoes (without any overlap or tetrominoes sticking out).

2. A map of major cities and roads of Mathland is shown below.
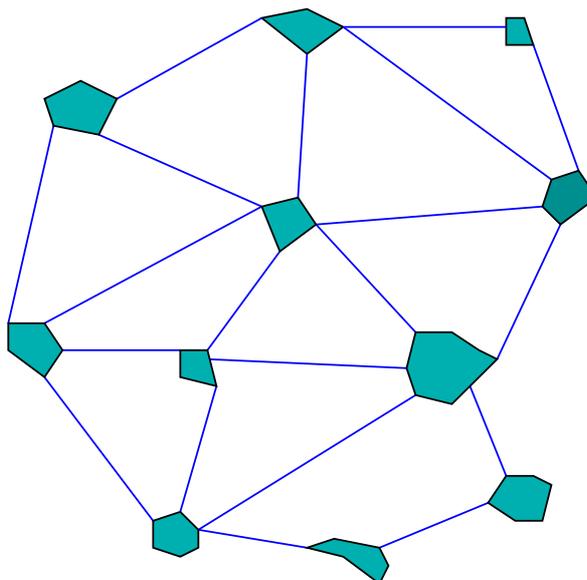
   (a) Is there a tour that starts and ends at the same city and uses each road exactly once?

   (b) Is there a tour that starts and ends at the same city and visits each of the other cities exactly once?

3. Two players play the following game. Turns alternate. In each move, a player can take 2, 3, or 4 chips from a pile that initially had 15 chips. The player that cannot make a move loses. Find a winning strategy for one of the players. (Describe your strategy and prove that it works. In particular, make it clear whether you should go first or second if you want to win.)

4. *ABCDEF* is a regular hexagon with side length 1 unit. Six circles with centers at its vertices and radii 1 unit are drawn as shown in the picture below. Find the (exact) area of the shaded "flower".
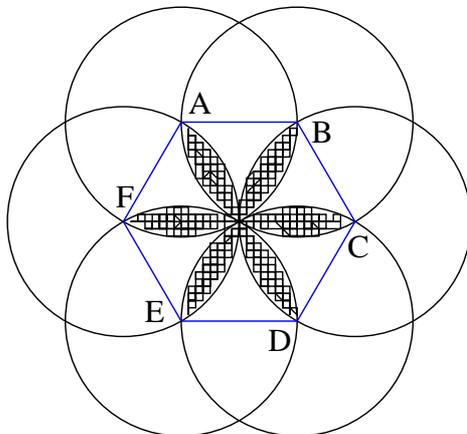
- **For extra credit:** The map below shows the cities and one-way roads in Sikinia.
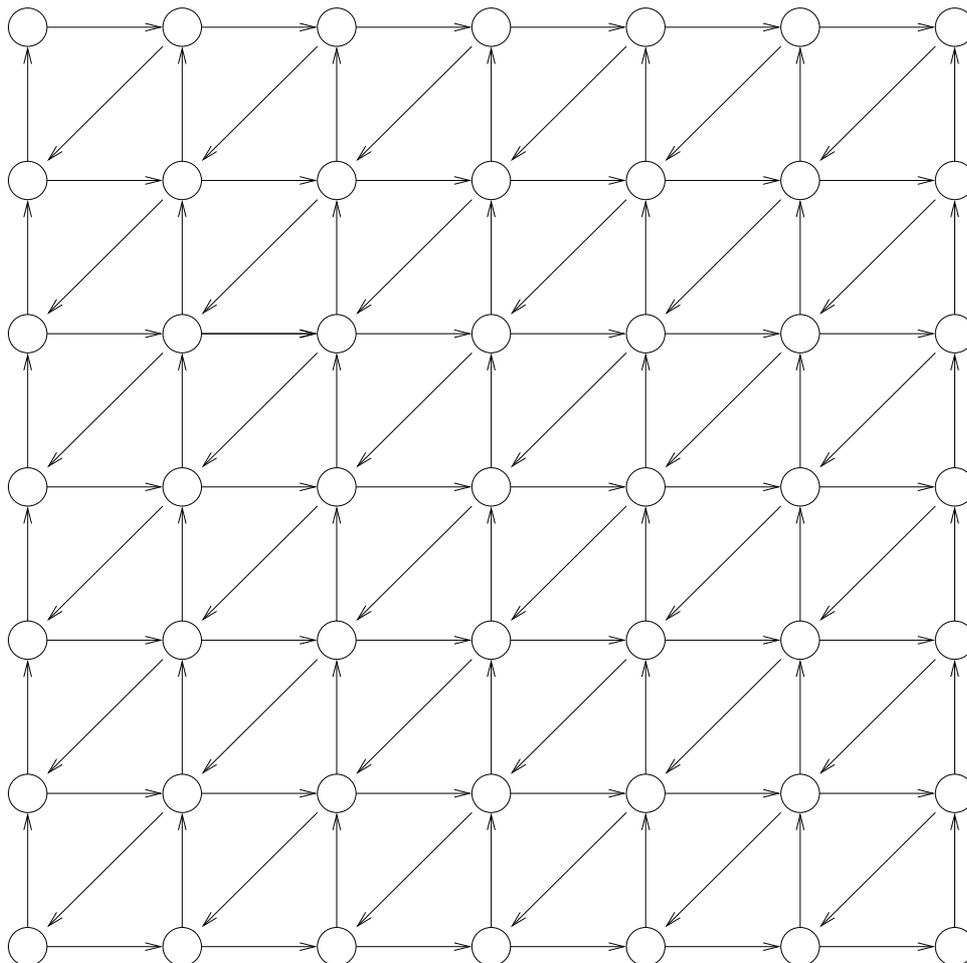
Is there a path, not necessarily closed, that visits every city exactly once?